MATHEMATICS EXTENSION 1

2010

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK 4

General Instructions

- Writing time –70 minutes.
- All three questions should be attempted
- Total marks available - 45
- All questions are worth 15 marks each
- An approved calculator may be used
- A table of standard integrals appears on the back page this half yearly examination
- All relevant working should be shown for each question. Start each of the three questions on a separate piece of writing paper.

Marks

QUESTION 1 (15 MARKS)

(a) Find the inverse of each of the following functions and state the domain and range of the inverse function

(i) $y = x^3$ 1

(ii) $y = \dfrac{1}{x+1}$ 1

(b) Evaluate $\sin^{-1}\left(-\dfrac{1}{2}\right) + \cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right) - \tan^{-1}\left(-\sqrt{3}\right)$ without the use of trigonometric functions on your calculator. 2

(c) Determine without a calculator $\cos\left[\tan^{-1}\left(\dfrac{4}{9}\right)\right]$ 2

(d) Find the general solution for $\cos\theta = \dfrac{1}{2}$ and find the solutions when $n = \pm 1$ 2

(e) Show that $y = \sqrt{x}$ and its inverse are mutually exclusive, i.e show that $f^{-1}[f(x)] = f[f^{-1}(x)] = x$ 2

(f) Differentiate with respect to x

(i) $y = \sin^{-1}(4x+1)$ 1

(ii) $y = 4\tan^{-1} 5x$ 1

(iii) $y = (\tan^{-1} x + 1)^5$ 1

(iv) $y = e^{\cos^{-1} x}$ 2

Marks

QUESTION 2 (15 MARKS)

(a) Find the integral (primitive function) of

(i) $\dfrac{1}{x^2+7}$ 1

(ii) $\dfrac{1}{\sqrt{4-x^2}}$ 1

(b) Find $\displaystyle\int \frac{dx}{\sqrt{5-13x^2}}$ 2

(c) Find $\dfrac{d}{dx}\left((x\tan^{-1}x - \dfrac{1}{2}\ln(x^2+1)\right)$ and hence evaluate $\displaystyle\int_0^1 \tan^{-1}x\,dx$ 3

(d) The curve $y = \dfrac{3}{\sqrt{x^2+4}}$ is rotated about the x - axis between 2

$x = 0$ and $x = 2$. Find the volume of the solid generated.

(e) For the function $f(x) = \sin x - \cos^2 x$

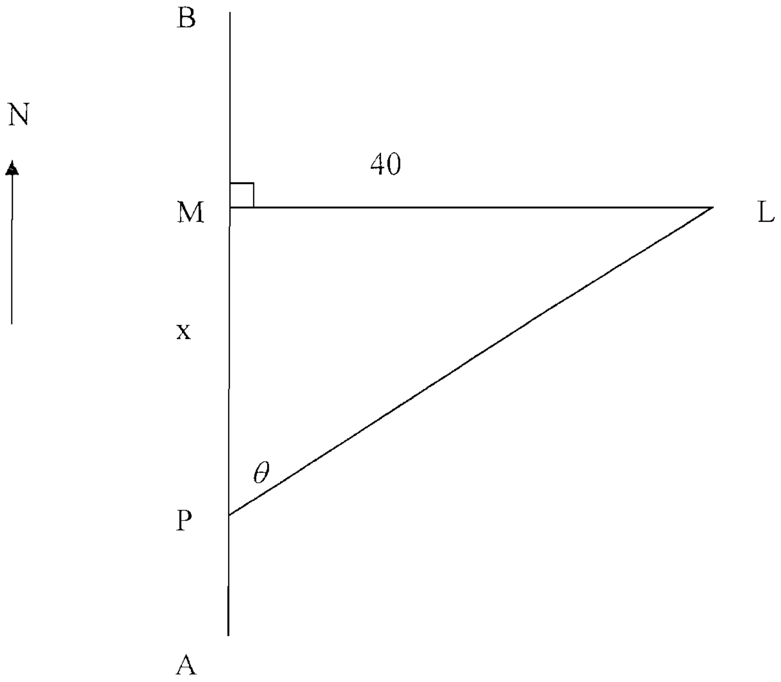
(i) Show that $f(x)$ has a root between $x = 2$ and $x = 3$. 1

(ii) Starting with $x_1 = 2.2$ use one application of Newton's method to find a better approximation for the root. Answer correct to 2 significant figures. 2

(f) The velocity of a particle is given by $v = 3x + 7\ cm\ s^{-1}$. 3
If the initial displacement is 1cm to the left of the origin, find the displacement as a function of time. (Hint : first find $\dfrac{dt}{dx}$)

QUESTION 3 (15 MARKS) **Marks**

(a)



A boat is sailing due North from point A to point B at a steady speed of $5\ ms^{-1}$. A marker buoy M on its route is situated 40 metres due West of a lighthouse L. When the boat is at point P at a distance of x metres from M, the bearing of the lighthouse from the boat is θ, $0 < \theta < \frac{\pi}{2}$.

(i) Show that $\theta = \tan^{-1}\frac{40}{x}$ 1

(ii) Hence find the rate at which θ is changing when $x = 20$ 3

Marks

(b) Consider the function $f(x) = \frac{1}{2}\cos^{-1}(1-3x)$.

(i) State the domain and range of $f(x)$. 2

(ii) Hence sketch the graph of $y = f(x)$. 2

(c) (i) Show that $f(x) = e^x - x^3 + 1$ has a zero between 4.4 and 4.6. 1

(ii) Find an approximation; correct to 1 decimal place, for this zero using the method of halving the interval. 2

(d) A bottle of medicine which is initially at a temperature of 10° C is placed into a room which has a constant temperature of 25° C. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine. That is, T satisfies the equation

$$\frac{dT}{dt} = -k(T-25)$$

i) Show that $T = 25 + Ae^{-kt}$ satisfies this equation. 1

ii) If the temperature of the medicine after ten minutes is 16° C, find its temperature after 40 minutes. 3

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$