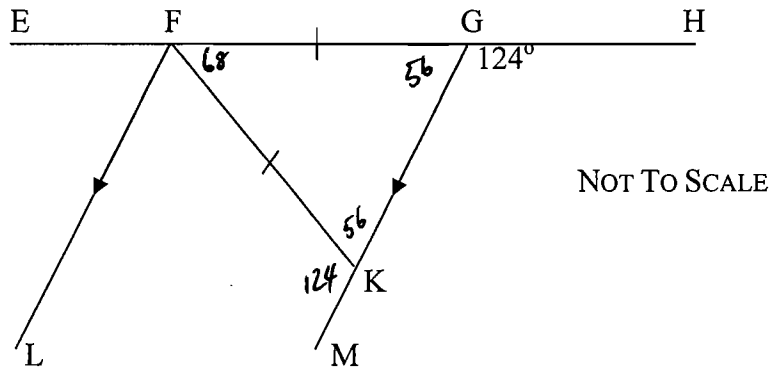


(a) In the figure below,  $FL \parallel GM$  and  $FG = FK$ .



(i) Prove that  $\angle FKG = 56^\circ$

2

(ii) Hence or otherwise prove that  $FL$  bisects  $\angle EFK$ .

2

(b) Find :

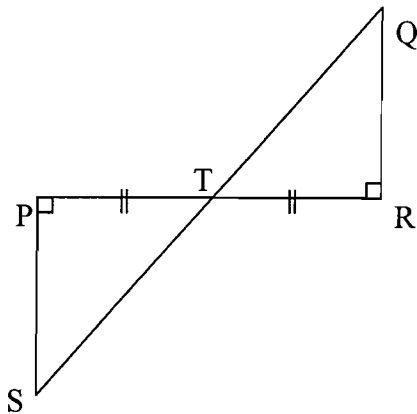
(i)  $\int \frac{3x^4 - 2x}{x^2} dx$

2

(ii)  $\int_0^{\frac{\pi}{2}} 2 \sin(3x) dx$

2

(c) In the diagram below,  $\angle SPT = \angle TRQ = 90^\circ$  and  $PT = TR$ .



(i) Prove that  $\triangle SPT \cong \triangle TRQ$

3

(ii) Prove that  $T$  bisects  $QS$ .

1

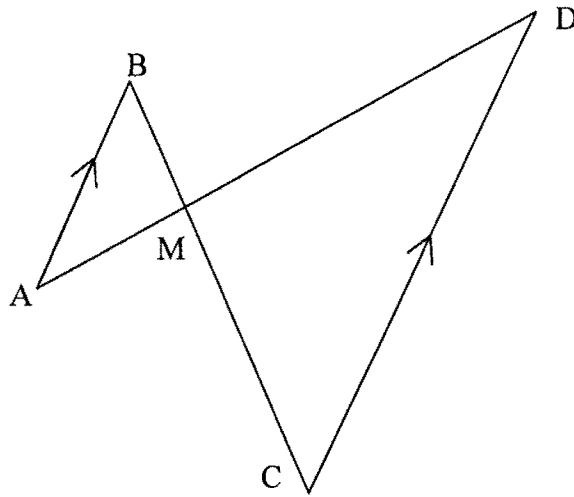
(a) The probability that a man lives to the age of 75 is  $\frac{3}{5}$  and the probability that his wife will live to the age of 75 years is  $\frac{2}{3}$ . By drawing a tree diagram or otherwise, find the probability that

- |      |  |   |
|------|--|---|
| i.   | both will live to the age of 75                  | 2 |
| ii.  | Only the man will live to the age of 75          | 1 |
| iii. | Only the wife will live to the age of 75         | 1 |
| iv.  | At least one of them will live to the age of 75. | 2 |

(b) If  $\alpha$  and  $\beta$  are the two roots of  $x^2 - 4x + 2 = 0$ , find the value of:

- |      |                                 |   |
|------|---------------------------------|---|
| i.   | $\alpha + \beta$                | 1 |
| ii.  | $\alpha\beta$                   | 1 |
| iii. | $\alpha^2\beta + \alpha\beta^2$ | 2 |
| iv.  | $\alpha^2 + \beta^2$            | 2 |

- (a) Consider the function defined by  $f(x) = 2x^3 - 3x^2 - 36x + 26$ .
- (i) Find the coordinates of the stationary points of the curve  $y = f(x)$  and determine their nature. **3**
- (ii) Find the coordinates of any point of inflexion. **1**
- (iii) Sketch the graph of  $f(x) = 2x^3 - 3x^2 - 36x + 26$  by showing the above information. **2**
- (iv) For what values of  $x$  is the curve concave down and decreasing? **2**
- (b) For the parabola  $4x = 8y - y^2$ .
- (i) Find the coordinates of the vertex. **2**
- (ii) Find the coordinates of the focus. **1**
- (iii) Sketch the curve clearly labelling the vertex and focus. **1**



- (a) The figure ABCD has  $AB \parallel CD$ . It also has the feature that DA intersects BC at M.
- (i) Prove  $\triangle AMB \parallel \triangle DMC$ . 3
- (ii) If  $AB:CD = 2:5$  and  $\text{area } \triangle AMB = 10u^2$ , find the total area of the figure. 1
- (b) (i) Find  $\int \cos(4x) dx$  1
- (ii) Evaluate  $\int_1^{e^4} \frac{x}{x^2 + 4} dx$  2
- (c) On the Cartesian Plane, sketch the region satisfying the inequalities  $x \geq 2$ ,  $y \geq 4$  and  $y \leq 8 - x$  3
- (d) A hat contains 3 white marbles, 4 black marbles, 9 red marbles and 4 green marbles. 2 marbles are drawn out without replacement. What is the probability that they are both red? 2

(a) The equation of a parabola is given by  $x^2 - 4x - 2y + 8 = 0$

- |          |      |   |   |
|----------|------|---|---|
| Find the | i.   | Vertex  | 2 |
|          | ii.  | Focus   | 2 |
|          | iii. | Equation of the normal to the parabola at the point (0, 4). | 2 |

(b) A woman walks 120 metres on a bearing of  $312^\circ$ , then turns and walks for a further 96 metres on a bearing of  $056^\circ$ .

- |     |  |   |
|-----|--|---|
| i.  | How far is the woman from her starting point to the nearest kilometre? | 2 |
| ii. | Hence find the bearing of the woman from her starting point?           | 2 |

(c) Use the table

$x$	3	3.25	3.5	3.75	4	4.25	4.5	2
$fx$	1.0	0.8	0.65	0.55	0.5	0.48	0.45	

to find an approximation to the value of the definite integral

$$\int_3^{4.5} f(x)dx,$$

using Simpson's Rule. Give your answer correct to 3 significant figures.

## Question 5 (12 marks) Begin a SEPARATE sheet of paper

(a) Show that  $\frac{\sec \theta - \sec \theta \cos^4 \theta}{1 + \cos^2 \theta} = \sin \theta \tan \theta$  3

(b) (i) Find the value(s) of  $k$  for which  $x^2 + (2 - k)x + 2.25 = 0$  has equal roots 2

(ii) Find the value(s) of  $k$  for which  $y = kx + 1$  is tangent to  $y = x^2 + 2x + 3 \cdot 25$  1

(c) A 15cm arc on the circumference subtends an angle of  $\frac{\pi^c}{5}$  at the centre of a circle. 3

Find the radius of the circle and the area of the sector.

(d) Boat A sails 15km from port P on a bearing of  $055^\circ$

Boat B sails from P for 25 km on a bearing of  $135^\circ$

• A

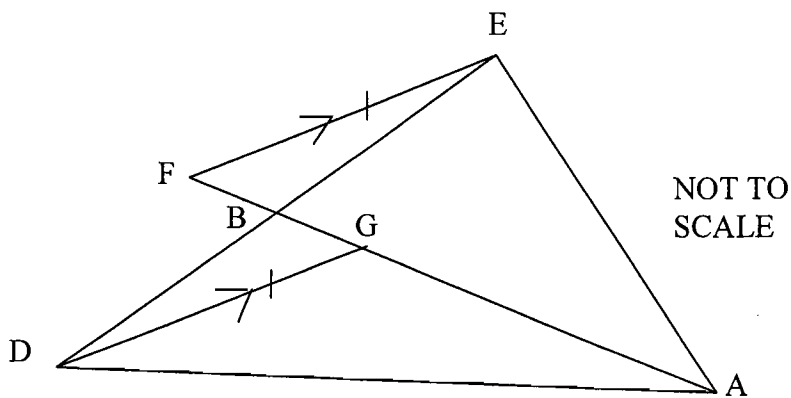
(i) Show the angle  $APB = 80^\circ$  1

• P

(ii) Calculate their distance apart to 1 dec pl. 2

• B

(a)



AB is a median in the triangle DEA i.e.  $BD=BE$ . Also  $FE=DG$  and FE is parallel to DG.

Copy the diagram into your answer booklet and prove, giving full reasons why  $FB = BG$

3

(b) There is one red and three green jellybeans in a jar. One jellybean is selected at random, eaten, and then a second jellybean is selected at random and is also eaten. Find the probability:

(i) The two jellybeans eaten are both green.

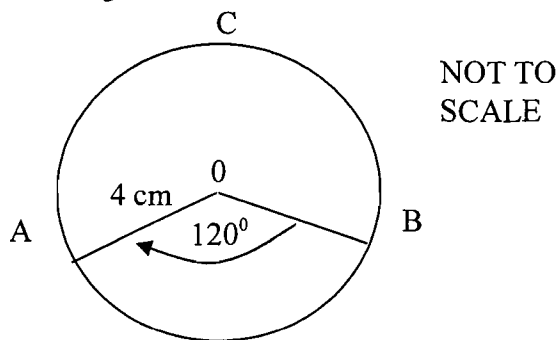
2

(ii) The red jellybean is the second one eaten.

1

(c) The circle shown has centre O, radius 4 cm and  $\angle AOB = 120^\circ$ . Show that the exact area of the major segment CAB is

$$\frac{32\pi+12\sqrt{3}}{3} \text{ cm}^2$$



3

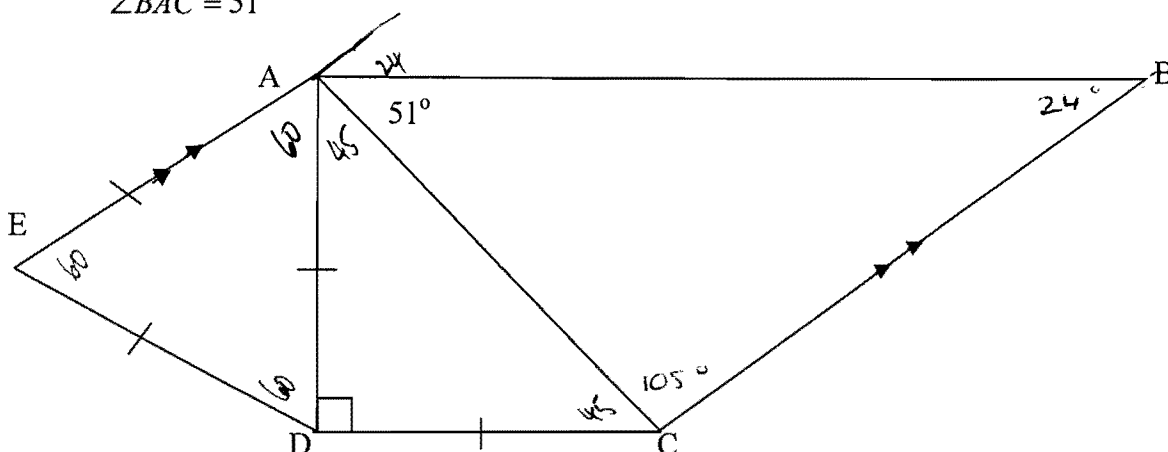
(d) (i) Sketch a neat graph of  $y = e^{2x} + 1$

1

(ii) Find the area bounded by  $y = e^{2x} + 1$ , the coordinate axes and the line  $x = 1$

2

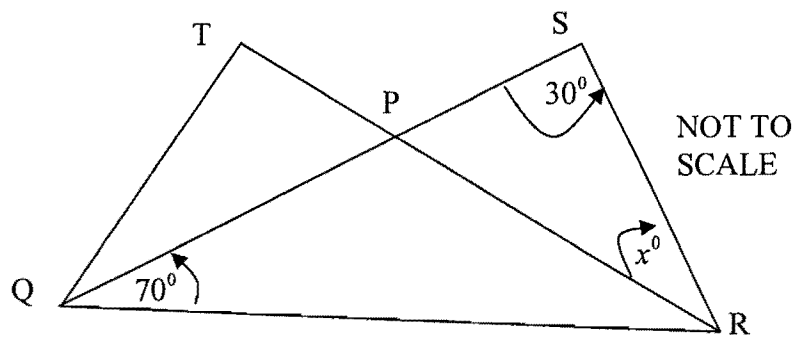
- a) In the diagram below  $AE = ED = AD = DC$ ,  $\angle ADC = 90^\circ$  and  $AE \parallel BC$ .  
 $\angle BAC = 51^\circ$



- i) Find the size of  $\angle EAB$ . Give reasons for your answer. 3
- ii) Find the size of  $\angle ABC$ . Give reasons for your answer. 1
- b) A particle moves in a straight line so that its displacement, in metres, is given by  
 $x = \frac{4t^2 + t + 8}{4t + 1}$  where  $t$  is measured in seconds.
- i) Find the initial position of the particle. 1
- ii) Find an expression for the velocity of the particle. 1
- iii) Show that the particle is stationary when  $t = \frac{-1 + 4\sqrt{2}}{4}$  seconds. 2
- iv) Describe the motion of the particle in the first two seconds. 2
- c) Solve the pair of simultaneous equations 2
- $$3x - y = 10$$
- $$x = y + 2$$



(a)



The equal sides QP and RP of the isosceles triangle QPR are produced to S and T respectively, such that PS=PT.

$$\angle PQR = 70^\circ, \angle PSR = 30^\circ \text{ and } \angle PRS = x^\circ$$

(i) Find the size of  $x$ . 2

(ii) Prove there is another angle equal to  $x^\circ$ . 2

(b) (i) If  $y = e^{2x^3}$ , find  $\frac{dy}{dx}$  2

(ii) Hence, or otherwise, evaluate  $\int_0^1 x^2 e^{2x^3} dx$  2

(c) Find the equations of the tangents to the parabola  $y = x^2 - 2x - 3$  at the points where the line  $y = 5$  cuts the parabola. 3

(d) Is the following series an arithmetic or geometric progression? Justify your answer.

$$\ln(x) + \ln(x^2) + \ln(x^3) + \ln(x^4) + \dots$$
1

- (a) In a raffle in which 1000 tickets are sold, there is a first prize of \$1000, a second prize of \$500 and a third prize of \$200. The prize winning tickets are drawn consecutively without replacement, with the first ticket winning first prize.

Find the probability that:

- i. a person buying one ticket in the raffle wins:

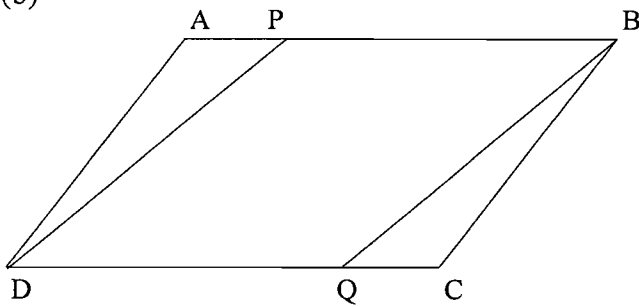
- α. first prize. 1
- β. at least \$500 1
- γ. no prizes. 1

- ii. a person buying two tickets in the raffle wins:

- α. at least \$500 1

(b)

3



ABCD is a parallelogram,  $BP = DQ$ .

Prove  $DP = BQ$

- (c) i. Is the series  $\log 3 + \log 9 + \log 27 + \dots$  arithmetic or geometric? 2  
Give reasons for your answer.

- iii. Find the sum of the first 10 terms of the series. 1

- (d) Find the radius and centre of the circle with equation 2

$$4x^2 - 4x + 4y^2 + 24y + 21 = 0$$