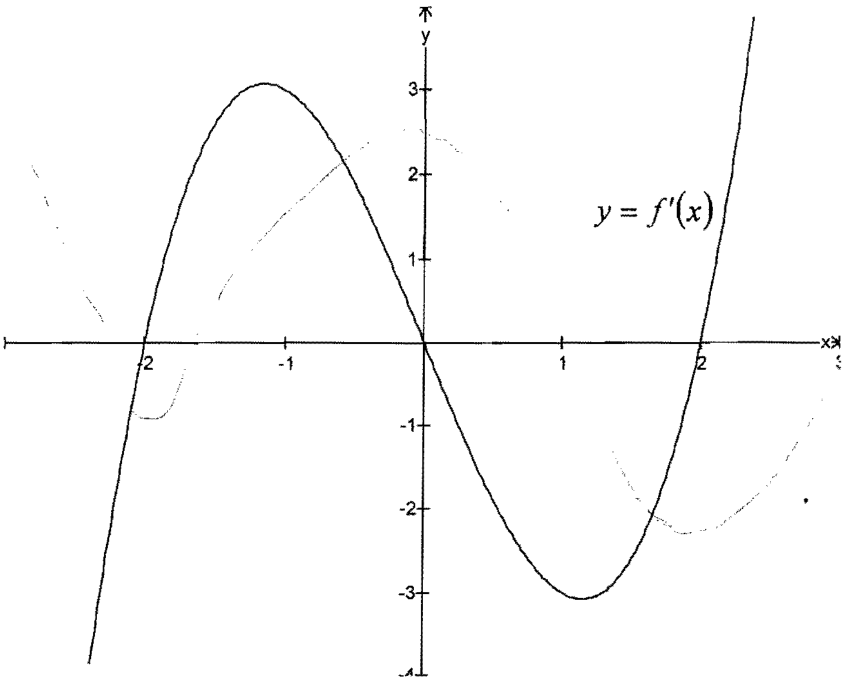
WR 2003

QUESTION 9 (12 MARKS) Use a SEPARATE Sheet of Paper

(a) The sketch below shows the graph of $y = f'(x)$ which is the *derivative* of a function $y = f(x)$.



(i) Give the x value(s) of all the turning points on the curve $y = f(x)$.

(ii) Give the x value(s) of the local maxima on the curve $y = f(x)$.

(iii)Give the x value(s) of any points of inflexion on the curve $y = f(x)$.

(iv)Give a possible sketch of $y = f''(x)$, the *second derivative* of $y = f(x)$.

QUESTION 9 CONTINUES ON THE NEXT PAGE

Marks

(b) Brad and Jennifer want to build up a deposit of \$50 000 to buy a house. They devise a savings plan to allow them to achieve this goal. Beginning on 1st January 2003, they deposit \$1000 on the first day of each month into an account which pays 9% p.a. compounded monthly.

(i) Show that the amount they have in the account at the end of January 2003 (i.e. at the end of 1 month) is given by $A_1 = 1000(1.0075)^1$. **1**

(ii) Show that the amount they have in the account at the end of February 2003 (i.e. at the end of 2 months) is given by $A_2 = 1000(1.0075^2 + 1.0075)$ **1**

(iii) Show that the amount they have in the account at the end of (n) months is given by $A_n = \dfrac{1007.5\left(1.0075^n - 1\right)}{0.0075}$ **2**

(iv) Hence find the least number of months they need to build up their deposit. **2**

QUESTION 10 (12 MARKS) Use a SEPARATE Sheet of Paper | **Marks**

(a) Consider the points A (-2,1) B (4,1) and P (x,y)

(i) Find expressions for the gradients of the two intervals PA and PB **1**

(ii) Find the equation of the locus of P if $\angle APB = 90^o$ **2**

(iii) Show that the locus represents a circle and give its' centre and radius. **2**

(b) For the quadratic expression: $2x^2 - x + 3$

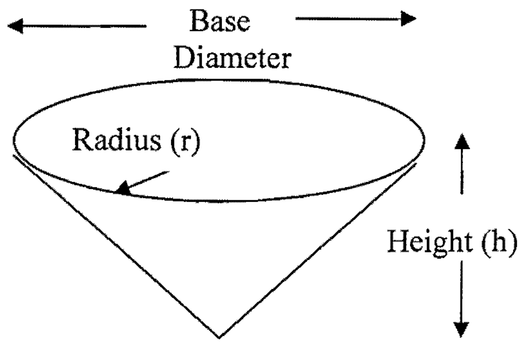
which has zeros when $x = \alpha$ and $x = \beta$.

(i) Show that the expression is positive definite. **1**

(ii) Find the value of $\alpha\beta$. **1**

(iii) Find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$. **1**

(c) The diagram below represents a conical water container. The sum of its base diameter and its height is 60 metres.



(i) Write an expression for the height in terms of the radius (r). **1**

(ii) Show that the volume is given by : $V = 20\pi r^2 - \dfrac{2}{3}\pi r^3$ **1**

(iii) Find the radius which makes the volume a maximum. **2**

WR 2004

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A function $f(x)$ is defined by the rule $f(x) = x^3 - 6x^2 + 9x - 5$ in the domain $0 \leq x \leq 4$.

i. Find the coordinates of the stationary points and determine their nature. **4**

ii. Find the coordinates of the point of inflexion. **2**

iii. Sketch the curve in the domain $0 \leq x \leq 4$. **2**

(b) Given that $\log_2 5 = 2.32$ and $\log_2 3 = 1.58$, find the value of:

i. $\log_2 15$ **1**

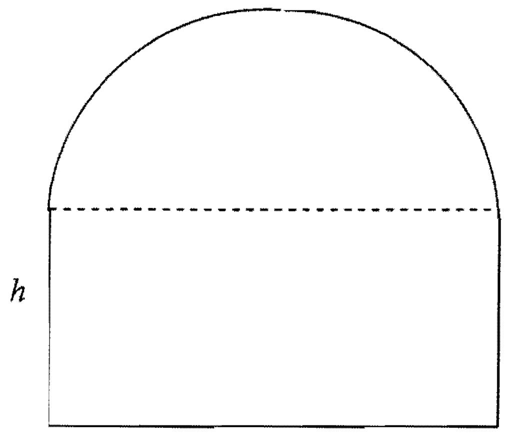
ii. $\log_2 0.6$ **1**

iii. $\log_2 60$ **2**

Question 10 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The population of a town has a growth rate proportional to the size of its population. $(i.e. \frac{dP}{dT} = kP)$. If the population of the town was 18 000 five years ago and it is currently 21 000, how long (to the nearest year) will it take the population of the town to reach 30 000 and what is the rate of change of population at that time? **5**

(b) Lisa has designed a garden bed which consists of a rectangle and a semicircle as shown in the diagram.



$2r$

If the perimeter of the garden bed is to be 20 metres:

i. Find an expression for h in terms of r. **1**

ii. Show that the area of the garden bed can be given by the formula $A = 20r - 2r^2 - \frac{1}{2}\pi r^2$. **1**

iii. Find the value of r that gives the maximum area and hence give this area to the nearest metre. **3**

(c) A curve has the following defining features. **2**

When x = -5, 0 and 3, $\frac{dy}{dx} = 0$.

$\frac{d^2y}{dx^2} > 0$ for $x < -2\frac{1}{2}$ and $x > 1\frac{1}{2}$, while

$\frac{d^2y}{dx^2} < 0$ for $-2\frac{1}{2} < x < 1\frac{1}{2}$.

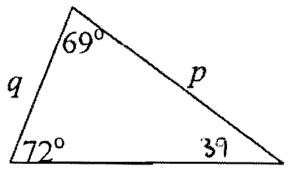
Give a possible sketch of the curve.

WR 2005

Question 9 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the sum of the first 100 multiples of 5 **2**

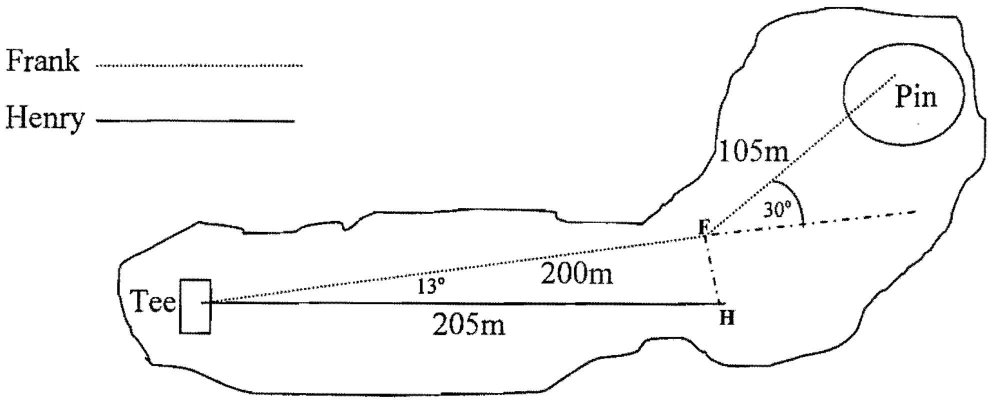
(b) Using the information in the diagram below to calculate the value of the ratio $\frac{p}{q}$ as a decimal correct to 3 decimal places **2**



(c) The 18th hole at Royal Maples is a dogleg to the left. Frank hits a 200m drive then turns left 30° and hits a 105m shot to the pin.

(i) What is the straight line distance from the tee to the pin? **2**

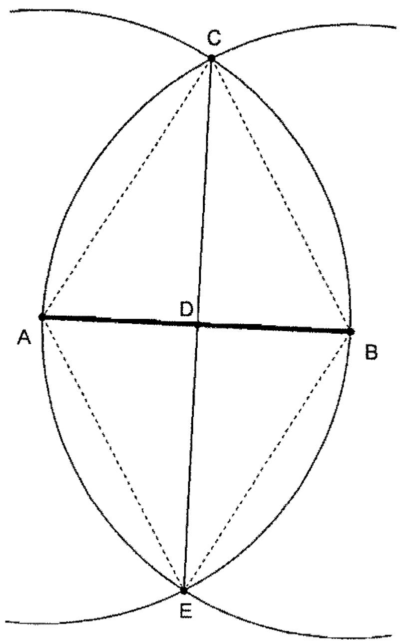
(ii) Henry hits his drive a distance of 205m and to the right of Frank's drive line by 13°. Show that the triangle formed by the two initial drives is approximately right angled. **2**



(d) Two circles of equal radius are drawn. One has it's centre at A and the other has it's centre at B.

(i) Prove $\Delta AEC \equiv \Delta BEC$ **2**

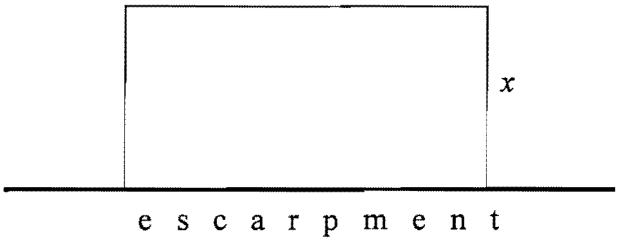
(ii) Hence show CE bisects AB **2**



Question 10 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) A farmer wishes to build a rectangular enclosure for his sheep.

Fortunately he can use a sandstone escarpment as one side of his rectangle.



He has sufficient fencing material for 200m of fence.

(i) If we let one side of the rectangle be x, write an expression for the area of the enclosure in terms of x. **2**

(ii) Find the maximum area enclosure the farmer can build. Be sure to justify that this area is a maximum. **3**

(b) A particle is moving in a straight line with velocity $v = 3e^{t} + 6e^{-t}$

It begins its motion at origin. t is measured in minutes and v in ms^{-1}.

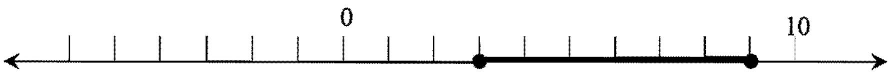
(i) What is the velocity initially? **1**

(ii) Find an equation for x, the displacement of the particle **2**

(iii) When $x = 10$ show that $3e^{2t} - 7e^{t} - 6 = 0$ **1**

(iv) Find t when $x = 10$ **1**

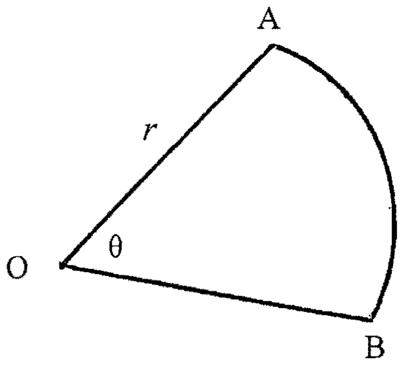
(c) The number line graph represents the solution to an inequality of the type $|x - a| \le b$. Find the value of a and b. **2**



WR 2006

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a)



The area of the sector AOB is 3π cm^2 while the length of AB is $\frac{3\pi}{2}$ cm. **3**

Find the value of θ and the radius, r.

(b) A ship is moving with an acceleration that is proportional to its velocity.

(i.e. $\frac{dv}{dt} = kv$)

If its initial velocity is 8m/s and 10 seconds later is 7 m/s, find:

i. the value of k to 3 significant figures and give the equation for the velocity of the ship. **2**

ii. the velocity of the ship after one minute. **1**

iii. the time at which the velocity is 2 m/s (to the nearest second) **2**

(c) If α and β are the roots of the equation $3x^2 - 2x - 4 = 0$, find:

i. $\alpha + \beta$ **1**

ii. $\alpha\beta$ **1**

iii. $(4 - \alpha)(4 - \beta)$ **2**

Question 10 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Peter deposits \$5000 at the end of each year into a superannuation fund with an interest rate of 10% p.a. compounded half yearly. He withdraws his investment immediately after making the 30th payment. **4**

What is the total of his investment?

(b) The area of a sector of a circle, radius r units, subtending an angle of θ at the centre is $100\ cm^2$.

i. Show that the perimeter of the sector can be given by **2**

$$P = 2r + \frac{200}{r}$$

i. Find the radius of the circle that makes the perimeter of the sector a minimum. **3**

(c) In a given state there are 1000 towns with postcodes ranging from 2000 to 2999 inclusive.

i. How many of these postcodes end in a zero (0)? **1**

ii. When addressing a letter to a friend who lives in this state, Sophie forgot the postcode, however she remembered that it ended with 0 and that at least two consecutive digits in the number are the same. **2**

What is the probability that the postcode is 2880?

WR 2007

Question 9 (12 marks) Begin a SEPARATE sheet of paper **Marks**

(a) Re-write $2y = x^2 - 6x + 8$ in the form $(x-h)^2 = 4A(y-k)$ **3**

Hence state the focus and vertex for the parabola

(b) The percentage concentration (A) of Carbon_{14} falls exponentially after the death of the living organism it is a part of. After 1845 years only 80% of the original concentration of Carbon_{14} remains.

(i) Using the model $A = 100e^{-kt}$, find the value of k **2**

(ii) Another organic artefact contains only 65% of the original concentration of Carbon_{14}. How long has this organism been dead? **2**

(iii) A sea sponge has been dead for 12 000 years. What percentage of the original Carbon_{14} concentration does it have? **2**

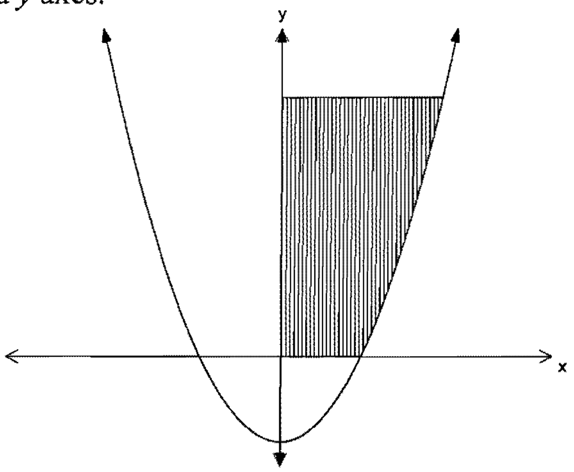
(c) On the same diagram sketch the graphs of $y = \sin x$ and $y = 2\sin x + 1$ **3**

$0 \le x \le 2\pi$

WR 2008

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes. **3**



Find the volume of the solid of revolution formed when the region is rotated about the y axis.

b) Paul plays computer games competitively. From past experience, Paul has a 0.8 chance of winning a game of *Beastie* and a 0.6 chance of winning a game of *Dragonfire*. In one afternoon of competition he plays two games of *Beastie* and one of *Dragonfire*.

i) What is the probability that he will win all three games? **1**

ii) What is the probability that he will win no games? **1**

iii) What is the probability that he will win at least one game? **1**

c) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of $\$M$. Let $\$A_n$ be the amount owing at the end of the nth month.

i) Find an expression for A_6. **1**

ii) Show that $A_8 = (20\,000 - 6M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$ **1**

iii) Find an expression for A_{36}. **2**

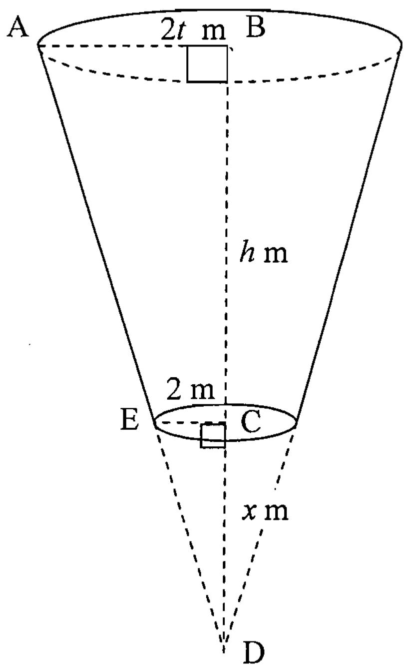
iv) Find the value of M. **2**

End of Question 9

Question 10 continued

Marks

b) A truncated cone is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The top radius is to be t times greater than the bottom radius which is 2 metres.



AB = $2t$ metres
BC = h metres
EC = 2 metres
CD = x metres

i) If x is the height of the removed section of the original cone, show using similar triangles that $x = \dfrac{h}{t-1}$ **2**

ii) Show that the volume of the truncated cone is given by $V = \left(\dfrac{4\pi h}{3}\right)\left(t^2 + t + 1\right)$ **2**

iii) If the upper radius plus the lower radius plus the height of the truncated cone must total 12 metres, calculate the maximum volume of the hopper. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A plant nursery has a watering system which repeatedly fills a storage tank then empties it's contents to water different sections of the nursery.
The volume of water (in cubic metres) in the tank at a time t is given by the equation

$$V = 2 - \sqrt{3}\cos t - \sin t \text{ where t is measured in minutes.}$$

i) Give an equation for $\frac{dV}{dt}$, the rate of change of the volume at a time t. **1**

ii) Is the tank initially filling or emptying? **1**

iii) At what time does the tank first become completely full and what is it's capacity when full? **3**

Question 10 continues on page 13

Question 10 (12 marks) Begin a SEPARATE sheet of paper

(a) Two sailors are paid to bring a motor launch back to Sydney from Gilligans Island, a distance of 1 200 km. They are each paid \$25 per hour for the time spent at sea.

The launch uses fuel at a rate $R = 20 + \frac{v^2}{10}$ litres per hour. Diesel costs \$1.25 per L and (v) is the velocity in km/hour.

(i) Show that, to bring the launch back from Gilligans Island, the total cost to the owners is $\frac{90000}{v} + 150v$. **3**

(ii) Find the speed which minimises the cost and determine this cost. **3**

(b) The sum of a geometric series is represented $\sum = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ **1**

Form an expression for $r\sum$ to show that $\sum = \frac{a(r^n - 1)}{r - 1}$

(c) (i) Paula is in a superannuation fund to which she contributes \$250.00 at the beginning of each month for 30 years. The fund pays 6·6% pa compounded monthly. If the fund matures at the end of the last month of the 30th year, find the total value of the fund at maturity. **3**

(ii) After receiving the payout from the fund, Paula sells her Audi for \$30 000 and invests the total of the two assets in an account that earns interest at 6.6% p.a. compounded monthly. How much will the investment be worth after a further 10 years? **2**