## WESTERN REGION Trial HSC Mathematics Solutions 2007

The methods of solution given are an indication only. Any reasonable approach should be accepted.

| Solutions | Marks/comment |
|---|---|
| 1. a) $3.718281828 \div 3\cdot141592654 = 1.184$ | **1** for division<br>**1** for rounding |
| b) $\sin\theta = 0\cdot500698846\ldots \quad \theta \cong 30^\circ$ | **1** evaluation<br>**1** angle |
| c) (-2, 3) is centre radius = 1.5 | **1** centre, **1** radius |
| d) $3 + 5 + 7 + x = 6\cdot75 \times 4 = 27$ whence $x = 12$ | **1** progress and **1** |
| e) $\lvert -3-4\times 2.35\rvert = \lvert -12.4\rvert = 12.4$ | **1** |
| f) $(3x+1)(x-2)$ | **2** allow 1 for progress |
| g) $2.12\times 180 \div \pi = 121.467\ldots = 121^\circ 28'$ | **1** **/12** |
| 2. a) i) $\dfrac{f'(x)}{f(x)} = \dfrac{2x-3}{x^2-3x}$ | **1** |
| ii) $\frac{1}{2}x^{-\frac{1}{2}}$ | **1** |
| iii) $\dfrac{vu'-uv'}{v^2} = \dfrac{2\cos x + \sin x(2x-1)}{\cos^2 x}$ | **2** allow 1 for progress |
| b) i) $2\sin\dfrac{x}{2} + c$ | **2** allow 1 for any expression with sin |
| ii) $\dfrac{1}{18}\displaystyle\int_0^1 3e^{3x}dx = \dfrac{1}{18}\left[e^{3x}\right]_0^1 = \dfrac{e^3-1}{18}\#\cong 1.0603$ (not req$^{\underline{d}}$) | **3** give 2 for correct process 1 error and 1 if 2 errors. |
| c) $y = \int 2\cos 2x dx = \sin 2x + c \quad$ If $x = \pi/4 \quad 3 = \sin \pi/2 + c \;=\; 1+2$<br>$y = \sin 2x + 2$ | **2** 1 for sin $2x$ 1 for $c$<br>**1** final form **/12** |
| 3. a) i) $AB = \sqrt{20} = 2\sqrt{5} \quad CD = \sqrt{45} = 3\sqrt{5}$ (simplification not needed) | **2** allow 1 for clear attempt using P'sT |
| ii) $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1} \qquad \dfrac{y-4}{x+3} = \dfrac{-1}{2}$ | **1** |
| $2y-8 = -x-3 \qquad x+2y-5+0$ | **1** |
| iii) $\dfrac{\lvert Ax_1+By_1+C\rvert}{\sqrt{A^2+B^2}} = \dfrac{\lvert 1\times 1+2\times -1-5\rvert}{\sqrt{1^2+2^2}} = \dfrac{6}{\sqrt{5}}\# = \dfrac{6\sqrt{5}}{5}$ | **1** sub form **1** value don't have to rationalise |
| iv) $\dfrac{6}{\sqrt{5}}\left(\dfrac{2\sqrt{5}+3\sqrt{5}}{2}\right) = 15\text{u}^2$ Adding ΔCBD & ΔABD = 9 + 6 = 15 | **1** |
| b) $y' = 3\cos 3x \quad f'(\pi/3) = 3\cos\pi = 3$ gradient of tangent | **1** |
| $f(\pi/3) = \sin\pi = 0 \qquad y_1$ at the tangent | **1** |
| $y - y_1 = m(x-x_1) \qquad y = 3x - \pi$ | **1** |
| c) if $0 > x^2+x+2$ then $-x^2+x-2 > 2x \quad -2 < x < 1$ | **2** allow 1 for prog **/12** |

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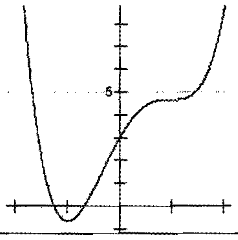
| 4. a) i) $(m^2-n^2)^2+(2mn)^2=m^4-2m^2n^2+n^4+4m^2n^2$ | **1** |
| $=m^4+2m^2n^2+n^4=(m^2+n^2)^2$ | **1** |
| ii) 60, 91, 109 | **1** |
| b) i) $y'=4x^3-4x^2-4x+4 \qquad y''=12x^2-8x-4$ | **1, 1** |
| ii) $f'(-1)=-4-4+4+4=0 \quad f'(1)=4-4-4+4=0$ | **1** |
| $f(-1)=1+\frac{4}{3}-2-4+3=-\frac{2}{3} \quad f(1)=1-\frac{4}{3}-2+4+3=4\frac{2}{3}$ | **1** |
| iii) $y''=0$ *if* $3x^2-2x-1=0=(3x+1)(x-1) \quad x=-\frac{1}{3}\, or\, 1$ | **1** |
| iv) $(1,4\frac{2}{3})$ is a H P of I since $f''(1)=0$ and gradient > 0 before & after | **1** |
| $(-\frac{1}{3},□1.5)$ is an inflexion since $f''(-\frac{1}{3})=0$ and gradient > 0 before & after | **1** |
| v) Main points calculations  labelled as per their | Award **2** if graph reflects their working<br>**/12** |
| LHS$=\dfrac{\sec\theta-\sec\theta\cos^4\theta}{1+\cos^2\theta}=\dfrac{\sec\theta(1-\cos^4\theta)}{1+\cos^2\theta}$ | **1** |
| 5 a) $=\dfrac{\sec\theta(1-\cos^2\theta)(1+\cos^2\theta)}{(1+\cos^2\theta)}=\sec\theta\sin^2\theta$ | **1** |
| $=\dfrac{1}{\cos\theta}\times\sin^2\theta=\dfrac{\sin\theta}{\cos\theta}\times\sin\theta=\sin\theta\tan\theta=\text{RHS}$ | **1** |
| b) i) $x^2+(2-k)x+2.25=0$ has $\Delta=(2-k)^2-4\times2.25=(2-k)^2-9$ | **1** |
| ie $2-k=\pm3$ *so* $k=-1$ *or* $5$ | **1** |
| ii) If $kx+1=x^2+2x+3.25$ then $0=x^2+(2-k)x+2.25$ | **1** |
| so tangent if $k$ = -1 or 5 | |
| c) $l=r\theta \quad 15=r\times\frac{\pi}{5} \quad r=\frac{75}{\pi}$ | **1** |
| $A=r^2\theta/2=\dfrac{75^2}{\pi^2}\times\dfrac{\pi}{10}=\dfrac{1125}{2\pi}\cong179.05$ | **2** |
| d) i) 135 -55 = 80 | **1** |
| ii) $p^2=a^2+b^2-2ab\cos P=25^2+15^2-2\times25\times15\times\cos80$ | **1** |
| $p^2=719.76... \quad p\cong26.8$ | **1** ignore rounding **/12** |
| 6 a) i) 2.25 + 1.75 = 4 | **1** |
| ii) 2.25 – 1.75 = 0.5 | **1** |
| iii) $1.75+2\times2.25=6.25 \quad 6.25\div5=1.25 \quad m^{1.25}$ | **1 + 1** |
| b) i) In the triangles ABD and CBD | |
| BD is common | |
| BA = BC *Equal radii* | **1** |
| AD = AC *Radii of circle and DC = AC Radii of circle* | **2** |
| $\therefore$ AD = DC | |
| $\Delta ABD\equiv\Delta CBD$ *SSS rule* | **1** |
| ii) $\angle ABD=\angle CBD$ *corresponding angles congruent triangles* | **1** justification required |
| Therefore $\angle ABC$ is bisected by DB | |

<table>
<tr><td>c) i) in a year there are 365 × 24 × 60 = 525600 minutes</td><td>1</td></tr>
<tr><td>$0.12 \div 525600 = 0.000000228$ and this amount plus 1 equals<br>1. 000000228</td><td>1</td></tr>
<tr><td>ii) $1000 \times 1.000000228^{525600} = \$1127.50$</td><td>1<br>/12</td></tr>
<tr><td>7 a) i) $m = \dfrac{y - y_1}{x - x_1}$ $and$ $(x_1, y_1) = (-2,0)$ $so$ $m_1 = \dfrac{y-0}{x--2} = \dfrac{y}{x+2}$</td><td>1</td></tr>
<tr><td>ii) likewise PB has gradient $\dfrac{y}{x-6}$ since PA is at right angles to PB</td><td>1</td></tr>
<tr><td>$\dfrac{y}{x+2} \times \dfrac{y}{x-6} = -1$ ie $y^2 = -x^2 + 4x + 12$</td><td>1</td></tr>
<tr><td>b) i) $x = \int vdt = 3t^2 - 8t - \dfrac{t^3}{3} + c$ but if $t = 0$ $x = 5$ so $x = 3t^2 - 8t - \dfrac{t^3}{3} + 5$</td><td>1 + 1 for finding $c$</td></tr>
<tr><td>ii) $v = 0 = (-4+t)(2-t)$ $ie$ $t = 4or2$</td><td>1</td></tr>
<tr><td>iii) $x = \left| \int_3^4 6t - 8 - t^2 dt \right| + \left| \int_4^5 6t - 8 - t^2 dt \right|$</td><td>1</td></tr>
<tr><td>$= \left| \left[ 3t^2 - 8t - t^3/3 \right]_3^4 \right| + \left| \left[ 3t^2 - 8t - t^3/3 \right]_4^5 \right| = \tfrac{2}{3} + 1\tfrac{1}{3} = 2\text{metres}$</td><td>1</td></tr>
<tr><td>iv) $a = 6 - 2t = 0\, if\ t = 3$ $v(3) = 18 - 8 - 9 = 1ms^{-1}$</td><td>1</td></tr>
<tr><td>c) i) $\tfrac{3}{8} \times \tfrac{3}{8} = \tfrac{9}{64}$</td><td>1</td></tr>
<tr><td>ii) $\tfrac{5}{8} \times \tfrac{5}{8} = \tfrac{25}{64}$</td><td>1</td></tr>
<tr><td>iii) $1 - (\tfrac{9}{64} + \tfrac{25}{64}) = \tfrac{30}{64} = \tfrac{15}{32}$</td><td>1 /12</td></tr>
<tr><td>8 a) i) $f(x)$increasing where $f'(x) > 0$ ie $-2 < x < 1/2$ and $x > 3$</td><td>1</td></tr>
<tr><td>ii) $f'(x)$ has a maximum so $f''(x) = 0$</td><td>1</td></tr>
<tr><td>C represents a point of inflexion on $f(x)$. gradient of $f(x) > 0$ before and after C</td><td>1</td></tr>
<tr><td>iii) $f(x)$ will be concave down when $f'(x)$ is decreasing $-0.95 < x < 1.95$</td><td>1</td></tr>
<tr><td>b) i) $V = \pi \int_e^{3e} y^2 dx = \pi \int_e^{3e} (\ln x)^2 dx$</td><td>2</td></tr>
<tr><td>ii)
<table>
<tr><td>$x$</td><td>$e$</td><td>$2e$</td><td>$3e$</td></tr>
<tr><td>$\pi \times (f(x))^2$</td><td>3.14</td><td>9.01</td><td>13.84</td></tr>
</table>
</td><td>2</td></tr>
<tr><td>iii) $\dfrac{h}{3}\left(f(a) + 4f(\tfrac{a+b}{2}) + f(b)\right) = \dfrac{e}{3}\left(3.14 + 4 \times 9.01 + 13.84\right) = 48.04$</td><td>2 allow 2 for correct process with their numbers</td></tr>
<tr><td>c) Domain $-3 \le x \le 3$ Range $0 \le y \le 3$</td><td>2 /12</td></tr>
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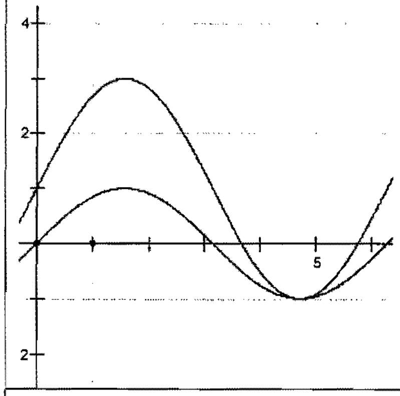
| 9 a) $2y = x^2 - 6x + 8$ $2y + 1 = x^2 - 6x + 9$ $4.\frac{1}{2}(y + \frac{1}{2}) = (x - 3)^2$<br>Thus the vertex is (3, -½) and the focus is (3, 0)<br>b) i) $t = 1845$ $A = 80$ $80 = 100e^{-1845k}$ $so$ $0 \cdot 8 = e^{-1845k}$<br>$\ln 0.8 = -1845k$ $k = 0.000120945$<br>ii) $65 = 100e^{-0.0001209845t}$ $\frac{\ln 0.65}{-0.000120945} = t \cong 3560 years$<br>iii) $A = 100e^{12000\times -0.000120945} = 23.4\%$<br>c)<br><br> | **1**<br>**2**<br>**1**<br>**1**<br>**1** for the sub + **1**<br>**1** for the sub +**1**<br>**3** Allow 1 or 2 for progress. Don't allow 3 for domain expressed in degrees or not close to the required restrictions |
| 10 a) i) Time to complete the trip = $\frac{1200}{v}$ and sailors paid \$50/hr thus<br>$Cost = \left(20 + \frac{v^2}{10}\right) \times \frac{1200}{v} \times 1.25 + 50 \times \frac{1200}{v}$<br>$= \frac{1200}{v}\left(75 + \frac{1.25v^2}{10}\right) = \frac{90000}{v} + 150v$<br>ii) $\frac{dCost}{dv} = 150 - \frac{90000}{v^2} = 0$ when $v^2 = 600$ $v = 24.495 km/hr$<br>$\frac{d^2c}{dv^2} = 180000v^{-3}$ at $v = 24.495$<br>$= \frac{180000}{24.495^3}$ >0 $\therefore$ min<br>then $Cost = \frac{90000}{24.495} + 150 \times 24.495$=\$7348.47<br>b) i)<br>$\Sigma = a + ar + ar^2 + ... + ar^{n-1}$ $r\Sigma = ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n$<br>$r\Sigma - \Sigma = ar^n - a$ $ie$ $\Sigma(r-1) = a(r^n - 1)$ $ie$ $\Sigma = \frac{a(r^n - 1)}{r - 1}$ | **1**<br>**1**<br>**1**<br>**1**<br>**1** determine min pt<br>**1**<br>**1** |

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| c) i) $6.6\% \div 12 = 0.55\% = 0{\cdot}0055 \quad r = 1.0055$ | 1 |
| First \$250 becomes $250 \times 1.0055^{360}$ last \$250 becomes $250 \times 1.0055$ | |
| Gp with $a = 250 \times 1.0055,\ n = 360,\ r = 1.0055 \quad \frac{a(r^n - 1)}{r - 1} =$ | 1 |
| $\frac{250 \times 1.0055(1.0055^{360} - 1)}{0.0055} = \$283\ 530.74$ | 1 |
| ii) With the Audi money $P = \$313\ 530.74 \quad r = 1.0055 \quad n = 120$ | 1 |
| $Pr^n = \$605\,520.87$ | 1 /12 |