2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

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Begin each question on a SEPARATE sheet of paper. Extra paper is available.

Marks

Question 1 (12 marks) Begin a SEPARATE sheet of paper

(a) Evaluate $\frac{e+1}{\pi}$ correct to three decimal places **2**

(b) Find θ to the nearest degree if $\sin\theta = \frac{4\sin 57^\circ}{6\cdot 7}$ **2**

(c) What is the centre and radius of a circle with equation $(x+2)^2 + (y-3)^2 = 2.25$ **2**

(d) The mean of 3, 5, 7, x is 6·75. What is the value of x? **2**

(e) If $x = 2\cdot 35$ evaluate the expression $|-3-4x|$ **1**

(f) Factorise $3x^2 - 5x - 2$ **2**

(g) Express $2\cdot 12^c$ as an angle in degrees correct to the nearest minute **1**

Question 2 (12 marks) Begin a SEPARATE sheet of paper **Marks**

(a) Differentiate

(i) $\log_e(x^2 - 3x)$ **1**

(ii) $x^{\frac{1}{2}}$ **1**

(iii) $\dfrac{2x-1}{\cos x}$ **2**

(b) Integrate

(i) $\int \cos\left(\dfrac{x}{2}\right)dx$ **2**

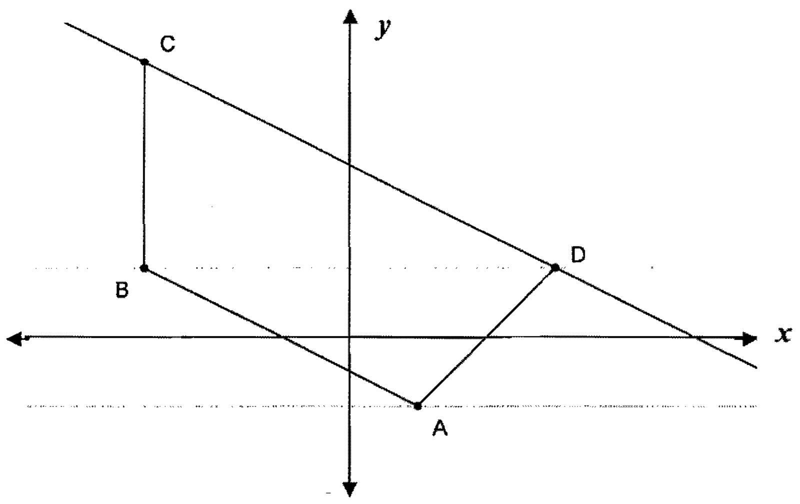
(ii) $\int_0^1 \dfrac{1}{6}e^{3x}\,dx$ leave your answer in exact from **3**

(c) Find y if $\dfrac{dy}{dx} = 2\cos 2x$ and $y = 3$ when $x = \dfrac{\pi}{4}$ **3**

Marks

Question 3 (12 marks) Begin a SEPARATE sheet of paper

(a) A (1, -1) B (-3, 1) C (-3, 4) and D (3, 1) are points on the Cartesian Plane. AB||CD



(i) Find the distances AB and DC **2**

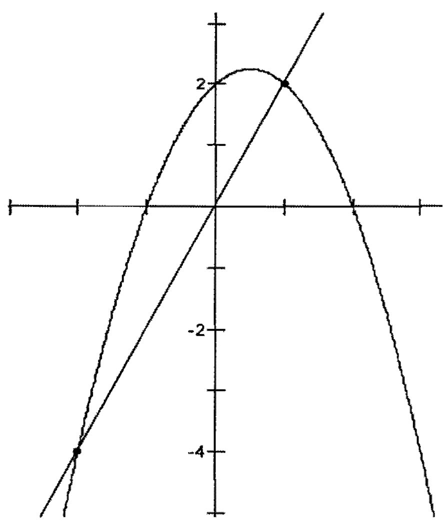
(ii) Show that the equation of CD is $x+2y-5=0$ **2**

(iii) Find the perpendicular distance of A from CD **2**

(iv) Hence or otherwise obtain the area of the trapezium ABCD **1**

(b) Find the equation of the tangent to the curve $y=\sin 3x$ at the point where $x=\frac{\pi}{3}$ **3**

(c) The graphs of $y=2x$ and $y=-x^2+x-2$ are shown. Solve $0>x^2+x+2$ **2**



Marks

Question 4 (12 marks) Begin a SEPARATE sheet of paper

(a) Let m and n be positive whole numbers with $m > n$

(i) Show that $m^2 + n^2$, $m^2 - n^2$, $2mn$ obey Pythagoras' Theorem **2**

(ii) Which Pythagorean Triad is generated when $m = 10$ and $n = 3$? **1**

(b) Consider the curve $y = x^4 - \frac{4}{3}x^3 - 2x^2 + 4x + 3$

(i) Obtain y' and y'' for this function **2**

(ii) Show that $x = -1$ and $x = 1$ satisfy $y' = 0$ and find the y coordinates. **2**

(iii) Find the x coordinates of the two points of inflexion. **1**

(iv) Determine the nature of each of the stationary points. **2**

(v) Sketch the curve for the domain $-2 \leq x \leq 2$ **2**

Question 5 (12 marks) Begin a SEPARATE sheet of paper

(a) Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$ **3**

(b) (i) Find the value(s) of k for which $x^2 + (2-k)x + 2.25 = 0$ has equal roots **2**

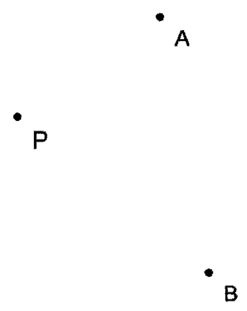
(ii) Find the value(s) of k for which $y = kx + 1$ is tangent to $y = x^2 + 2x + 3\cdot 25$ **1**

(c) A 15cm arc on the circumference subtends an angle of $\dfrac{\pi^c}{5}$ at the centre of a circle. **3**

Find the radius of the circle and the area of the sector.

(d) Boat A sails 15km from port P on a bearing of 055^0

Boat B sails from P for 25 km on a bearing of 135^0



(i) Show the angle APB = 80^0 **1**

(ii) Calculate their distance apart to 1 dec pl. **2**

Marks

Question 6 (12 marks) Begin a SEPARATE sheet of paper

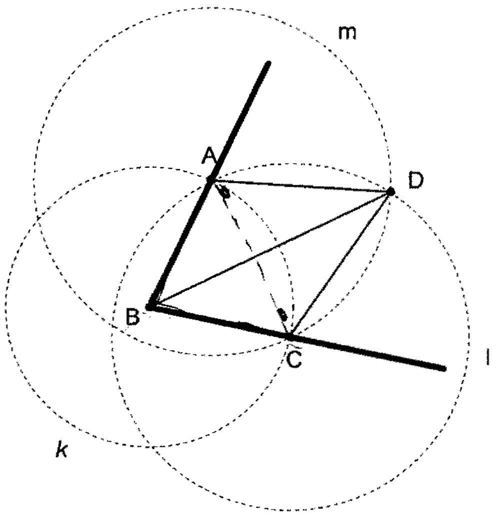
(a) $\log_m p = 1.75$ and $\log_m q = 2.25$. Find

(i) $\log_m pq$ 1

(ii) $\log_m \dfrac{q}{p}$ 1

(iii) $\sqrt[5]{pq^2}$ in terms of m 2

(b) In the diagram;
the circle k has centre B and radius BC.
the circle l has centre C and radius CA.
the circle m has centre A and radius AC.



(i) Prove $\Delta BAD \equiv \Delta BCD$ 4

(ii) Prove BD bisects $A\hat{B}C$ 1

(c) *Twinkle Finance* offers its investors the opportunity to have interest credited to their investment "as often as you wish". Naturally many investors punt for the "EVERY MINUTE" plan. *Twinkle* offer 12%pa.

(i) Stella invests \$1000 for a year with *Twinkle* on the "EVERY MINUTE" plan. Theoretically, *Twinkle's* computers multiply Stella's balance By approximately 1·000 000 228 every minute. Show why this is so. 2

(ii) How much is Stella's investment worth after 1 year? 1

Marks

Question 7 (12 marks) Begin a SEPARATE sheet of paper

(a) Let A be the point (-2, 0) and B be the point (6, 0).

At P (x, y), PA meets PB at right angles.

(i) Show that the gradient of PA is $m_1 = \dfrac{y}{x+2}$ **1**

(ii) Find an equation for the locus of P **2**

(b) The velocity of an object is given by the equation $v = 6t - 8 - t^2$

Where time (t) is in seconds and velocity (v) in metres/second

It begins its motion at $x = 5$ metres.

(i) Find an equation for the displacement of the object **2**

(ii) At what 2 times is the object stationary? **1**

(iii) Find the distance travelled by the object between $t = 3$ and $t = 5$ **2**

(iv) What is the maximum velocity of the object? **1**

(c) Two dice are biased so that, the probability of a six is $\dfrac{3}{8}$ and of each other number is $\dfrac{1}{8}$.

Find the probability of

(i) Rolling a double six **1**

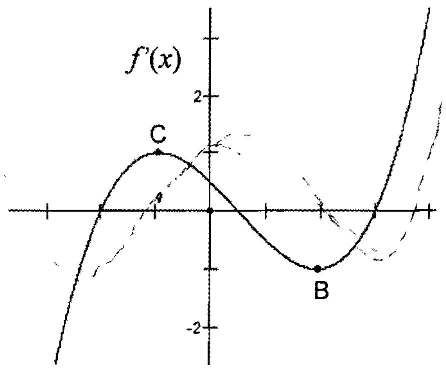
(ii) Rolling the two dice so that neither is a six **1**

(iii) Only 1 six appears when the two dice are rolled **1**

Question 8 (12 marks) Begin a SEPARATE sheet of paper **Marks**

(a) The graph of $y = f'(x)$ is shown. The roots of $f'(x)$ are $x = -2$, 0.5, and 3

C has x coordinate -0·95 and B has x coordinate 1·95



(i) For what values of x is $f(x)$ increasing? **1**

(ii) C is a local maximum on $f'(x)$. **2**

What type of point occurs on $f(x)$ at the same x value as that shown at C. Justify your answer.

(iii) For what values of x is $f(x)$ concave down? **1**

(b) The curve $y = \log_e x$ between $x = e$ and $x = 3e$ is rotated around the x axis.

(i) Write the integral which gives the value of this volume. **2**

(ii) Complete the table for this function write your answer to 2 decimal places **2**

x	e	$2e$	$3e$
$\pi \times (f(x))^2$			

(iii) Use Simpson's Rule with 3 function values to approximate the volume. **2**

(c) What is the domain and range for $y = \sqrt{9 - x^2}$ **2**

Question 9 (12 marks) Begin a SEPARATE sheet of paper **Marks**

(a) Re-write $2y = x^2 - 6x + 8$ in the form $(x-h)^2 = 4A(y-k)$ 3

Hence state the focus and vertex for the parabola

(b) The percentage concentration (A) of Carbon_{14} falls exponentially after the death of the living organism it is a part of. After 1845 years only 80% of the original concentration of Carbon_{14} remains.

(i) Using the model $A = 100e^{-kt}$, find the value of k 2

(ii) Another organic artefact contains only 65% of the original concentration of Carbon_{14}. How long has this organism been dead? 2

(iii) A sea sponge has been dead for 12 000 years. What percentage of the original Carbon_{14} concentration does it have? 2

(c) On the same diagram sketch the graphs of $y = \sin x$ and $y = 2\sin x + 1$ 3

$0 \le x \le 2\pi$

Marks

Question 10 (12 marks) Begin a SEPARATE sheet of paper

(a) Two sailors are paid to bring a motor launch back to Sydney from Gilligans Island, a distance of 1 200 km. They are each paid \$25 per hour for the time spent at sea.

The launch uses fuel at a rate $R = 20 + \frac{v^2}{10}$ litres per hour. Diesel costs \$1.25 per L and (v) is the velocity in km/hour.

(i) Show that, to bring the launch back from Gilligans Island, the total cost to the owners is $\frac{90000}{v} + 150v$. **3**

(ii) Find the speed which minimises the cost and determine this cost. **3**

(b) The sum of a geometric series is represented $\Sigma = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ **1**
Form an expression for $r\Sigma$ to show that $\Sigma = \frac{a(r^n - 1)}{r - 1}$

(c) (i) Paula is in a superannuation fund to which she contributes \$250.00 at the beginning of each month for 30 years. The fund pays 6·6% pa compounded monthly. If the fund matures at the end of the last month of the 30th year, find the total value of the fund at maturity. **3**

(ii) After receiving the payout from the fund, Paula sells her Audi for \$30 000 and invests the total of the two assets in an account that earns interest at 6.6% p.a. compounded monthly. How much will the investment be worth after a further 10 years? **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$