WESTERN REGION

2007
TRIAL HSC
EXAMINATION

Mathematics

Extension 1

SOLUTIONS

Question 1	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)	$y = \sin^{-1}\left(\frac{2x}{3}\right)$ $\frac{2x}{3} = \sin y$ $x = \frac{3}{2}\sin y$ Domain is $-\frac{3}{2} \le x \le \frac{3}{2}$ Range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	2	1 each for domain and range
(b)	Ends are at A(-4, 1) and B(x, y) P (-2, 5) divides AB in the ratio 2 : 3. $\frac{2x+3(-4)}{5} = -2$ and $\frac{2y+3(1)}{5} = 5$ $2x-12=-10$ $\quad$ $2y+3=25$ $2x=2$ $\quad$ $2y=22$ $x=1$ $\quad$ $y=11$ B is the point (1, 11)	2 marks	1 mark for equations 1 for solution
(c)	Using $u = 2x^2 - 3x$ find $\int \frac{(4x-3)}{\sqrt{2x^2-3x}}$ $u = 2x^2 - 3x$ $\frac{du}{dx} = 4x - 3$ $du = (4x-3)\,dx$ $\int \frac{(4x-3)dx}{\sqrt{2x^2-3x}} = \int \frac{du}{\sqrt{u}}$ $= \int u^{-\frac{1}{2}} du$ $= 2u^{\frac{1}{2}} + c$ $= 2(2x^2-3x)^{\frac{1}{2}} + c$ $= 2\sqrt{2x^2-3x} + c$	3 marks	3 marks for final solution. 2 marks if small error made in any stage but other wise okay 1 mark if du found or a start made

Question 1	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(d)	$x=\sin\theta$ and $y=\cos^2\theta-3$ $y=\left(1-\sin^2\theta\right)-3$ $y=\left(1-x^2\right)-3$ $y=-2-x^2$	2 marks	2 marks for correct solution. 1 mark if method correct, but single error made.
(e)	$\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{3x}{4}\right)}{2x}=\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{3x}{4}\right)}{\frac{8}{3}\times\frac{3\times 2x}{8}}$ $=\dfrac{3}{8}\times\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{3x}{4}\right)}{\frac{3x}{4}}$ $=\dfrac{3}{8}$	3 marks	3 marks for final solution. 2 marks if small error made in any stage but other wise okay 1 mark if an attempt made to get standard limit

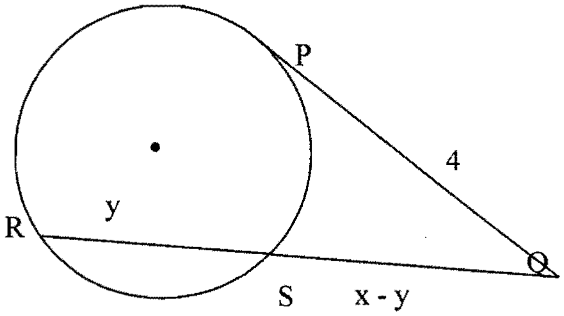
Question 2	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)	$\frac{d}{dx}\left(x\cos^{-1}x\right)=x.\left(\frac{-1}{\sqrt{1-x^2}}\right)+1.\cos^{-1}x$ $=\frac{-x}{\sqrt{1-x^2}}+\cos^{-1}x$	2	1 for use of product rule 1 for individual derivatives
(b)	Required term of $\left(\frac{1}{3}x^2+2\right)^5$ is $\binom{5}{4}\left(\frac{1}{3}x^2\right)^4(2)^1=\frac{10}{81}x^8$ Required coefficient is $\frac{10}{81}$	2	2 for correct result 1 if state the general term correctly but don't simplify.
(c)	$\int_0^{\frac{\pi}{3}}\sec 2x\tan 2x\ dx=\left[\frac{1}{2}\sec 2x\right]_0^{\frac{\pi}{3}}$ $=\left(\frac{1}{2}\sec\frac{2\pi}{3}\right)-\left(\frac{1}{2}\sec 0\right)$ $=-1-\frac{1}{2}$ $=-1\frac{1}{2}$	2	1 for correct use of standard integrals 1 for substitution
(d) i)	$T=25+Ae^{-kt}\Rightarrow T-25=Ae^{-kt}$ $\frac{dT}{dt}=-kAe^{-kt}$ $\frac{dT}{dt}=-k(T-25)$	1	Mark only if deriviative found and result shown

Question 2	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(d) ii)	$T = 25 + Ae^{-kt}$ When $t = 0$, $T = 10$ $10 = 25 + A(1)$ $A = -15$ When $t = 10$, $T = 16$ $16 = 25 - 15e^{-10k}$ $\frac{9}{15} = e^{-10k}$ $\ln\left(\frac{9}{15}\right) = -10k$ $k = 0.051$ (2 sig fig) When $t = 40$ $T = 25 - 15e^{-0.051(40)}$ $= 23$ (2 sig fig) The temperature is about $23°C$	3	3 marks for obtaining final answer. 2 marks if error made in calculation of A or of k. 1 mark if only A is found or if method is correct, but there are multiple errors..
(e)	$\int \cos^2 9x \, dx = \int \frac{1}{2}(1 + \cos 18x)\, dx$ $= \frac{1}{2}\left(x + \frac{1}{18}\sin 18x\right) + c$ $= \frac{x}{2} + \frac{1}{36}\sin 18x + c$	1 1	

Question 3	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(a)i)	$f(x) = \sin x - \cos^2 x$ has a root between $x = 2$ and $x = 3$ if it changes sign. $f(2) = \sin 2 - \cos^2 2$ ≈ 0.74 (2 sig fig) $f(3) = \sin 3 - \cos^2 3$ ≈ -0.84 (2 sig fig) So a root exists between $x = 2$ and $x = 3$	1	
a)ii)	$f(x) = \sin x - \cos^2 x$ $f'(x) = \cos x - 2\cos x.(-\sin x)$ $= \cos x + 2\cos x \sin x$ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= 2.2 - \frac{f(2.2)}{f'(2.2)}$ $= 2.2 - \frac{\sin 2.2 - \cos^2 2.2}{\cos 2.2 + 2\cos 2.2 \sin 2.2}$ $= 2.2 - (-0.30)$ $= 2.5$ (to 2 sig fig)	3	1 for derivative 1 for correct use of Newtons Method 1 for evaluating
(b)	Arrangements $= \frac{8!}{3!2!} = \frac{40320}{12} = 3360$ ways	2	1 for 8! 1 for division
(c)	Probability of more than two faulty grommets = 1 – P(two or less faulty) =1- [P(0 f) + P(1 f) +P(2 f)] $= 1 - [\binom{10}{0}(0.09)^0(0.91)^{10} + \binom{10}{1}(0.09)^1(0.91)^9 + \binom{10}{2}(0.09)^2(0.91)^8]$ $= 1 - [0.95]$ $= 0.05$	 1 1	

Question 3	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(d) (i)	$x = 4\cos\left(2t - \frac{\pi}{6}\right)$ $\dot{x} = -8\sin\left(2t - \frac{\pi}{6}\right)$ $\ddot{x} = -16\cos\left(2t - \frac{\pi}{6}\right)$ $\ddot{x} = -4\left[4\cos\left(2t - \frac{\pi}{6}\right)\right]$ $\ddot{x} = -2^2\left[4\cos\left(2t - \frac{\pi}{6}\right)\right]$ $\ddot{x} = -2^2 x$ which is of the form $\ddot{x} = -n^2 x$ so it is in simple harmonic motion.	2	1 for $\ddot{x}$ 1 for statement of SHM
(d) ii)	Amplitude is 4 units	1	
(d) iii)	Maximum speed is 8 m/s	1	

Question 4	HSC Trial Examination- Extension 1		2007
Part	**Solution**	**Marks**	**Comment**
(a)	Prove $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{n(n+1)}=\frac{n}{n+1}$ Assume for $n=k$ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{k(k+1)}=\frac{k}{k+1}$ Show that when $n=k+1$ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$ $LHS=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k(k+2)+1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ $=\frac{k+1}{k+2}$ $=RHS$ $\therefore$ if true for $n=k$, is also true for $n=k+1$ When $n=1$ LHS $=\frac{1}{1(1+1)}=\frac{1}{2}$ RHS $=\frac{1}{1+1}=\frac{1}{2}$ $\therefore$ true for $n=1$, and by induction true for all integers $n\geq1$	3	1 mark for stating the assumption. 1 for proving case for k+1 1 for n=1 and conclusion. Adjust accordingly if done in different order.

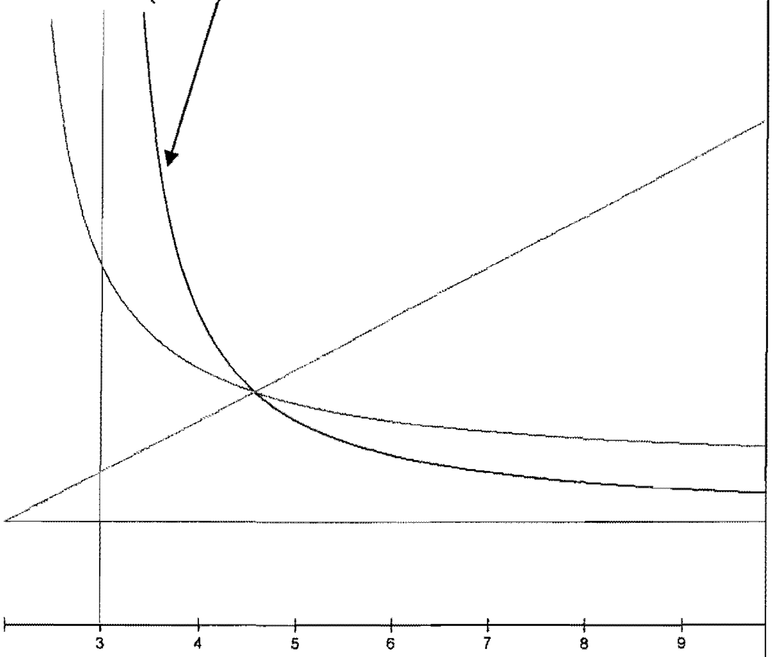
Question 4	HSC Trial Examination- Extension 1		2007
Part	**Solution**	**Marks**	**Comment**
(b)	a) In the circle centre O, the tangent PQ is 4 cm. The secant RQ is x cm and the chord RS is y cm.  (i) $PQ^2 = RQ \,.\, QS$ $4^2 = x\,(x-y)$ $16 = x^2 - xy$ $xy = x^2 - 16$ $y = x - \frac{16}{x}$	1	
	(ii) $y = x - 16x^{-1}$ $\frac{dy}{dx} = 1 + 16x^{-2}$ $= 1 + \frac{16}{x^2}$ As $x^2 \geq 0$, $\frac{dy}{dx} > 0$ $\therefore$ y is an increasing function for $x > 0$	2	Other statements about graph which show it is increasing are acceptable.
	(iii) If $x = 4$ then $y = 0$ This means that QS = RQ =PQ =4 RQ (SQ) becomes a tangent to the circle. (Tangents from an external point are equal)	1	Either statement relating to both lines being tangents okay for the mark.
(c) i)	$\alpha + \beta + \gamma = \frac{-b}{a} = 2$	1	
(c) ii)	$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = 4$ $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 2^2 - 2(4)$ $= -4$	1	

Question 4	HSC Trial Examination- Extension 1		2007
Part	**Solution**	**Marks**	**Comment**
(c) iii)	$\alpha\beta\gamma = \frac{-d}{a} = 5$ $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma} = \frac{4}{5}$	1	
(d)	Amplitude $= a = 3$ Period $= 4\pi = \frac{2\pi}{n} \rightarrow n = \frac{1}{2}$ $v^2 = n^2\left(a^2 - x^2\right)$ $v^2 = \left(\frac{1}{2}\right)^2\left(3^2 - x^2\right)$ $v^2 = \frac{\left(9 - x^2\right)}{4}$	2	1 for correct values of a and n 1 for equation

Question 5	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(a) i)	$P(X=5)=\binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3=\frac{1792}{6561}=0.273$	1	Full mark if left as product.
(a) ii)	$P(X\geq 8)=\binom{10}{8}\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^2+\binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1+\binom{10}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^0$ $=0.299$	2	2 marks if left as sum of terms
(b)	$\angle SQP=90^\circ$ (angle in a semicircle) $\angle TPS=x^\circ$ (alternate angles on || lines) $\angle TSQ=\angle TSP+x^\circ$ (adjacent angles) $\angle QPT=90^\circ+x^\circ$ (adjacent angles) $\angle TSQ+\angle QPT=180^\circ$ (opposite angles in cyclic quadrilateral) $\angle TSP+x^\circ+90^\circ+x^\circ=180^\circ$ $\angle TSP=90^\circ-2x^\circ$	3	Alternate solutions possible. 3 marks for complete solution 2 marks if a step is missing 1 if a start made with a correct relevant statement
(c)	$\sin 5x=\sin(4x+x)$ $=\sin 4x\cos x+\cos 4x\sin x$ $=2\sin 2x\cos 2x\cos x+(\cos^2 2x-\sin^2 2x)\sin x$ $=4\sin x\cos x(\cos^2 x-\sin^2 x)\cos x+\left((\cos^2 x-\sin^2 x)^2-(2\sin x\cos x)^2\right)\sin x$ $=4\sin x\cos^4 x-4\sin^3 x\cos^2 x+\cos^4 x\sin x-2\sin^3 x\cos^2 x+\sin^5 x-4\sin^3 x\cos^2 x$ $=5\sin x\cos^4 x-10\sin x^3\cos^2 x+\sin^5 x$	3	Three marks for any form that includes only powers of sinx & cosx 2 marks for incomplete expansion 1 mark if started using any valid breakup of sin5x

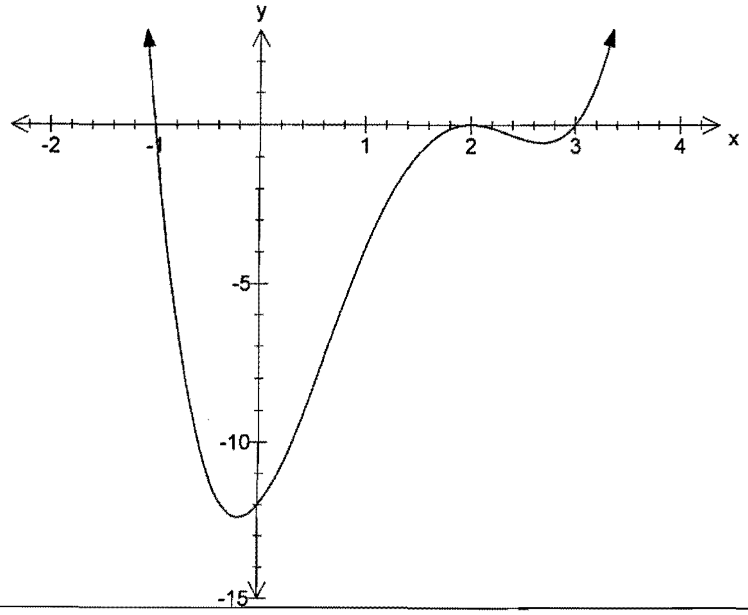
Question 5	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(d) i)	$\tan 20^\circ = \dfrac{h}{XZ}$ $\quad \tan 28^\circ = \dfrac{h}{YZ}$ $XZ = \dfrac{h}{\tan 20^\circ}$ $\quad YZ = \dfrac{h}{\tan 28^\circ}$	1	1 mark if both expressions given
(d) ii)	$XZ^2 + YZ^2 = 500^2$ $\dfrac{h^2}{\tan^2 20^\circ} + \dfrac{h^2}{\tan^2 28^\circ} = 500^2$ $h^2\left(\dfrac{\tan^2 28^\circ + \tan^2 20^\circ}{\tan^2 28^\circ \tan^2 20^\circ}\right) = 250000$ $h^2 = \dfrac{250000\tan^2 28^\circ \tan^2 20^\circ}{\tan^2 28^\circ + \tan^2 20^\circ}$ $h^2 = 22551.44$ $h = 150$ m	2	2 marks for use of Pythagoras and final answer. 1 mark if started using Pyth or trig correctly, but not finished

Question 6	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a) i)	$x = 240t\cos\theta \qquad x = 3600$ $y = 2000 + 240t\sin\theta - gt^2 \qquad y = 3200 - gt^2$ To intercept, the x and y values must be equal. $240t\cos\theta = 3600$ and $2000 + 240t\sin\theta - gt^2 = 3200 - gt^2$ $t\cos\theta = 15$ and $t\sin\theta = 5$ $\frac{t\sin\theta}{t\cos\theta} = \frac{1}{3}$ $\tan\theta = \frac{1}{3}$ $\theta = 18°26'$ $t = \frac{5}{\sin\theta} = \frac{5}{\sin 18°26'} = 15.8$ sec	3	3 for full solution obtained 2 if equated x and y and attempted to solve 1 if equated x and y only
ii)	$y = 3200 - gt^2$ $= 3200 - 10 \times 15.8^2$ $= 700$ metres	1	
(b) i)	$(1+x)^{n+1} = \binom{n+1}{0} + \binom{n+1}{1}x + \binom{n+1}{2}x^2 + \binom{n+1}{3}x^3 + \ldots\ldots$ $(1+x)(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots.$ $+\binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \binom{n}{3}x^4 + \ldots.$ $= \binom{n}{0} + \left(\binom{n}{1} + \binom{n}{0}\right)x + \left(\binom{n}{1} + \binom{n}{2}\right)x^2 + \ldots.$ Equating coefficients of x^2 $\binom{n+1}{2} = \binom{n}{1} + \binom{n}{2}$	2	2 for full solution 1 if wrote out expansion but not equated coeff or mistake in expansion

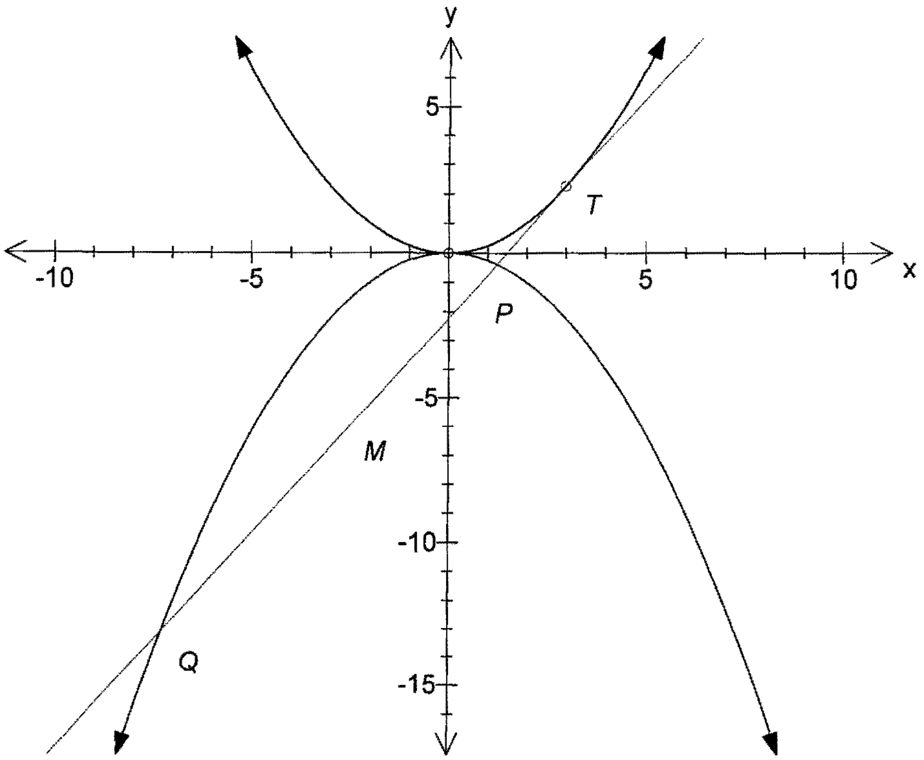
Question 6	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(b) ii)	$(1+x)^{2n}=\binom{2n}{0}+\binom{2n}{1}x+\binom{2n}{2}x^2....\binom{2n}{2n-1}x^{2n-1}+\binom{2n}{2n}x^{2n}$ Differentiating both sides gives: $2n(1+x)^{2n-1}=\left(\binom{2n}{1}+2\binom{2n}{2}x+....(2n-1)\binom{2n}{2n-1}x^{2n-2}+(2n)\binom{2n}{2n}x^{2n-}\right.$ Let $x=1$ $2n(2)^{2n-1}=\left(\binom{2n}{1}+2\binom{2n}{2}+....(2n-1)\binom{2n}{2n-1}+(2n)\binom{2n}{2n}\right)$ $n(2)^{2n}=\binom{2n}{1}+2\binom{2n}{2}+3\binom{2n}{3}....(2n-1)\binom{2n}{2n-1}+(2n)\binom{2n}{2n}$ $n4^n=\binom{2n}{1}+2\binom{2n}{2}+3\binom{2n}{3}....(2n-1)\binom{2n}{2n-1}+(2n)\binom{2n}{2n}$	2	2 for full solution 1 if differentiation done correctly but not finished or mistake in diff then followed on okay
(c)	$f(x)=2+\dfrac{4}{(x-3)}$  	2	1 for sketch 1 for asymptotes x = 3 and y = 2

Question 6	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(c) ii)	Inverse function comes from $x=2+\frac{4}{y-3}$ $x-2=\frac{4}{y-3}$ $\frac{y-3}{4}=\frac{1}{x-2}$ $y-3=\frac{4}{x-2}$ $y=3+\frac{4}{x-2}$ Inverse function is $f^{-1}(x)=3+\frac{4}{x-2}$ Sketch of inverse shown but not required.	2	2 for full solution 1 if substituted x and y correctly but mistake made after

Question 7	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(a) i)	20 rev/min $= 20 \times 2\pi$ rad/min $= \frac{40\pi}{60} = \frac{2\pi}{3}$ rad/sec	1	
(a) ii)	$\tan\theta = \frac{x}{8}$ $x = 8\tan\theta$ $\frac{dx}{d\theta} = 8\sec^2\theta$ $\frac{dx}{dt} = \frac{dx}{d\theta}.\frac{d\theta}{dt}$ $= 8\sec^2\theta.\frac{2\pi}{3}$ $= \frac{16\pi}{3}\sec^2\theta$ $= \frac{16\pi}{3}(1+\tan^2\theta)$ $= \frac{16\pi}{3}(1+\tan^2\theta)$ $= \frac{16\pi}{3}\left(1+\left(\frac{x}{8}\right)^2\right)$	2	2 marks for full solution 1 mark if done in terms of θ or otherwise incomplete
(a) iii)	At A, $x = 0$ $\frac{dx}{dt} = \frac{16\pi}{3}\left(1+\left(\frac{0}{8}\right)^2\right)$ $= \frac{16\pi}{3}$ At B, $x = 12$ $\frac{dx}{dt} = \frac{16\pi}{3}\left(1+\left(\frac{12}{8}\right)^2\right)$ $= \frac{16\pi}{3}\left(\frac{13}{4}\right)$ $= \frac{52\pi}{3}$ Difference $= \frac{52\pi - 16\pi}{3}$ $= \frac{9\pi}{4}$ m/s	2	1 mark if only one found correctly or if subtraction incorrect.

Question 7	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(b) i)	$P(x)=x^4+Ax^3+9x^2+4x-12=0$ $P(3)=3^4+A.3^3+9.3^2+4.3-12=0$ $27A+162=0$ $27A=-162$ $A=-6$	1	
(b) ii)	Sum of roots $=\frac{-b}{a}=\frac{-(-6)}{1}=6$ $(3)+(-1)+2\gamma=6$ $2\gamma=4$ $\gamma=2$ Roots are 3, -1, 2, 2 $P(x)=(x-3)(x+1)(x-2)^2$	1	
(b) iii)		2	1 mark for correct roots including double root 1 mark for correct orientation and y intercept

Question 7	HSC Trial Examination- Extension 1	2007

Part	Solution	Marks	Comment
(c)		3	Diagram not needed.

Tangent has equation $y = tx - at^2$

Intersects 2nd parabola where

$x^2 = -4a\left(tx - at^2\right)$

$x^2 + 4atx - 4a^2t^2 = 0$

$$x = \frac{-4at \pm \sqrt{(4at)^2 - 4.1\left(-4a^2t^2\right)}}{2}$$

$$x = \frac{-4at \pm \sqrt{16a^2t^2 + 16a^2t^2}}{2}$$

$$x = \frac{-4at \pm \sqrt{32a^2t^2}}{2}$$

$$x = \frac{-4at \pm 4\sqrt{2}at}{2}$$

$$x = \frac{4at\left(-1 \pm \sqrt{2}\right)}{2}$$

$$y = t\left(\frac{-4at \pm 4\sqrt{2}at}{2}\right) - at^2$$

$y = -2at^2 \pm 2\sqrt{2}at^2 - at^2$

$y = -3at^2 \pm 2\sqrt{2}at^2$

Comment: 2 for coordinates of end points of PQ

Question 7	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
	Find coordinates of M $x=\dfrac{\dfrac{4at\left(-1+\sqrt{2}\right)}{2}+\dfrac{4at\left(-1-\sqrt{2}\right)}{2}}{2}$ $x=-2at\rightarrow t=\dfrac{x}{-2a}$ $y=\dfrac{-3at^2+2\sqrt{2}at^2+-3at^2-2\sqrt{2}at^2}{2}=-3at^2$ $y=-3a\left(\dfrac{x}{-2a}\right)^2$ $y=-\dfrac{3x^2}{4a}$ Which is a parabola.		1 for equation of locus.