2007

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 2 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (84)

- **Attempt Questions 1-7**
- **All questions are of equal value**

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) State the domain and range of $y = \sin^{-1}\left(\frac{2x}{3}\right)$. **2**

b) The point P (-2, 5) divides the interval joining A (-4, 1) and B (x, y) internally in the ratio 2 : 3. Find the coordinates of the point B. **2**

c) Using the substitution $u = 2x^2 - 3x$, or otherwise, find $\int \frac{(4x-3)dx}{\sqrt{2x^2 - 3x}}$ **3**

d) Find the Cartesian equation of the curve defined by the parametric equations $x = \sin\theta$ and $y = \cos^2\theta - 3$ **2**

e) Evaluate $\lim_{x \to 0} \frac{\sin\left(\frac{3x}{4}\right)}{2x}$ **3**

End of Question 1

Question 2 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Find $\frac{d}{dx}\left(x\cos^{-1}x\right)$ **2**

b) Find the coefficient of x^8 in the expansion of $\left(\frac{1}{3}x^2+2\right)^5$ **2**

c) Use the table of standard integrals to evaluate $\int_0^{\frac{\pi}{3}} \sec 2x \tan 2x \; dx$ **2**

d) A bottle of medicine which is initially at a temperature of 10° C is placed into a room which has a constant temperature of 25° C. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine. That is, T satisfies the equation

$$\frac{dT}{dt}=-k(T-25)$$

i) Show that $T=25+Ae^{-kt}$ satisfies this equation. **1**

ii) If the temperature of the medicine after ten minutes is 16° C, find its temperature after 40 minutes. **3**

e) Find $\int \cos^2 9x \; dx$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) For the function $f(x) = \sin x - \cos^2 x$

i) Show that $f(x)$ has a root between $x = 2$ and $x = 3$. **1**

ii) Starting with $x_1 = 2.2$ use one application of Newton's method to find a better approximation for the root. Answer correct to 2 significant figures. **3**

b) How many distinct eight letter arrangements can be made using the letters of the word PARALLEL? **2**

c) The probability of a grommet being faulty after manufacture is 0.09. Find the probability that there are more than two faulty grommets in a batch of ten. **2**

d) A particle P is moving in a straight line with its position in metres from a fixed origin at a time t seconds being given by

$$x = 4\cos\left(2t - \frac{\pi}{6}\right).$$

i) Show that P is moving in simple harmonic motion. **2**

ii) What is the amplitude of the motion? **1**

iii) What is the maximum speed of the particle? **1**

End of Question 3

Marks

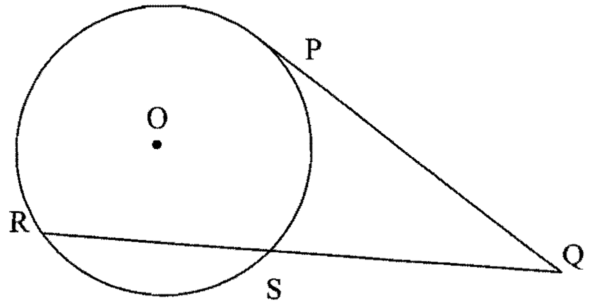
Question 4 (12 marks) Use a SEPARATE sheet of paper.

a) Use mathematical induction to prove that **3**

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots\ldots\frac{1}{n(n+1)}=\frac{n}{n+1}$$

for all positive integers *n*.

b) In the circle centre O, the tangent PQ is 4 cm. The secant RQ is x cm and the chord RS is y cm.



i) Show that $y = x - \frac{16}{x}$ **1**

ii) Show that as x increases, so does y. **2**

iii) What is the geometric significance of the case where $x = 4$? **1**

c) For the polynomial equation $p(x) = x^3 - 2x^2 + 4x - 5 = 0$, with roots $\alpha,\ \beta,$ and γ, find the value of

i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha^2 + \beta^2 + \gamma^2$ **1**

iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **1**

d) A particle is moving in simple harmonic motion about a fixed point O. Its period is 4π seconds and its amplitude is 3 m. Give an equation that relates its velocity v to its position x. **2**

End of Question 4

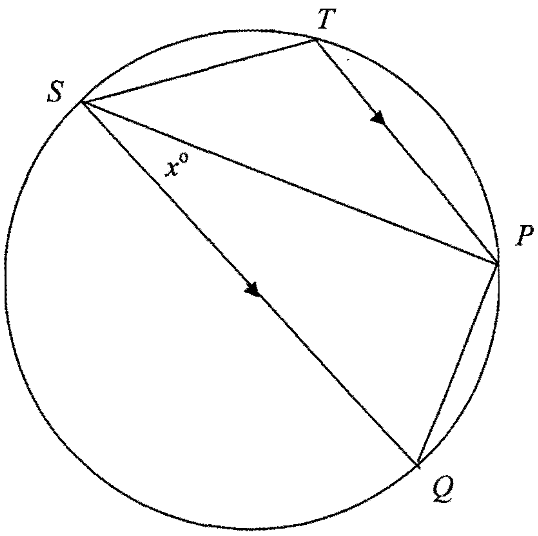
Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Experience shows there is a probability of $\frac{2}{3}$ that Josie will choose the winner of any one game in football tipping competitions.

i) In a football tipping competition in which there are 8 games in a round, what is the probability that she will pick 5 winners? **1**

ii) In a competition in which there are 10 games in a round what is the probability that she will pick at least 8 winners? **2**

b) The points P, Q, S and T lie on the circumference of a circle. SQ is a diameter of the circle and TP $\parallel$ SQ.
$\angle PSQ = x^o$ **3**
Find an expression for $\angle TSP$ in terms of x.

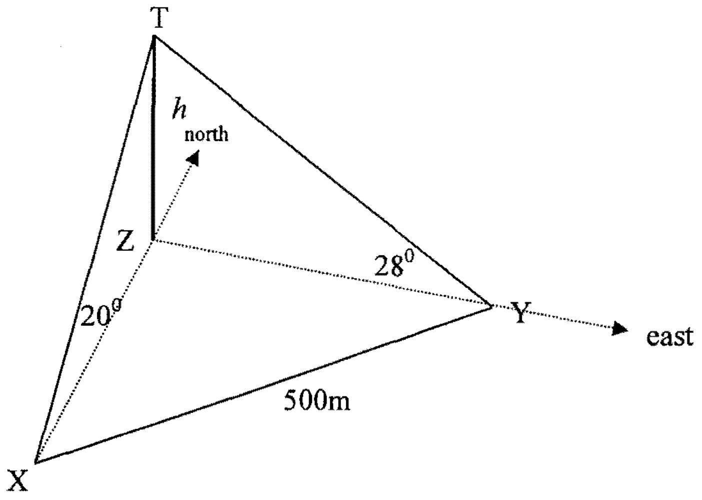


c) Find an expression for $\sin 5x$ in terms of $\sin x$ and $\cos x$ **3**

Question 5 continues on page 7

Question 5 (Continued) **Marks**

d) A conservationist observes the angle of elevation of the top of a tree, which is h metres tall, from two positions. From a point X, due south of the tree, it is 20° and from point Y, due east of the tree, it is 28°. The distance XY is 500 m.



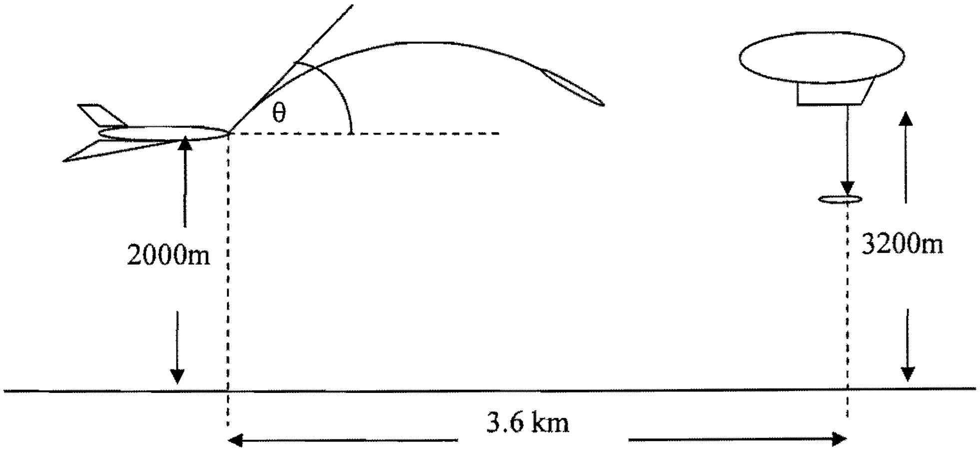
i) Write expressions for XZ and YZ in terms of h. **1**

ii) Calculate the value of h. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) A plane flying at a height of 2000 m observes a stationary blimp at a height of 3200 metres drop an object. The moment the object is released, the plane fires a projectile at an angle θ to the horizontal in the direction of the object at a velocity of 240 m/s. The horizontal distance between the plane and the blimp is 3.6 km at the time the projectile is fired.



The equations of motion of the projectile are :

$$x = 240t\cos\theta$$
$$y = 2000 + 240t\sin\theta - gt^2$$

The equations of motion of the dropped object (relative to a point below the plane) are :

$$x = 3600$$
$$y = 3200 - gt^2$$
(Use $g = 10\text{ms}^{-2}$)

i) What is the angle at which the projectile must be fired to intercept the object, and how long does it take to reach it? **3**

ii) At what height does the projectile intercept the object? **1**

Question 6 continues on page 9

Marks

Question 6 (Continued)

b) i) Use the expansion of the equation $(1+x)^{n+1} = (1+x)(1+x)^n$ to show that : **2**

$$\binom{n+1}{2} = \binom{n}{1} + \binom{n}{2}$$

ii) By differentiation of $(1+x)^{2n}$ show that **2**

$$\binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \ldots\ldots\ldots + n\binom{2n}{n} = n.4^n$$

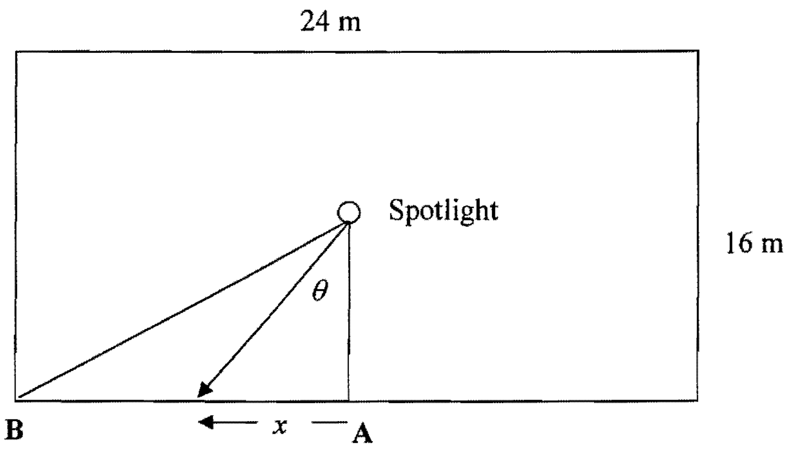
c) i) Sketch the function $f(x) = 2 + \frac{4}{(x-3)}$ for $x > 3$, indicating any asymptotes. **2**

ii) Find the inverse function $f^{-1}(x)$. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) A spotlight is in the centre of a rectangular nightclub which measures 24 m by 16 m. It is spinning at a rate of 20 rev/min. Its beam throws a spot which moves along the walls as it spins.



i) Write the rate of rotation $\frac{d\theta}{dt}$ in radians/sec **1**

ii) Find an expression for the velocity $\frac{dx}{dt}$ in terms of x at which the spot appears to be moving along the wall from A to B. **2**

iii) What is the difference in the velocities at which the spot appear to be moving at the points A, nearest to the light and B, furthest from the light? **2**

b) i) The polynomial $P(x) = x^4 + Ax^3 + 9x^2 + 4x - 12 = 0$ has a root at $x = 3$. Find the value of A. **1**

ii) The polynomial has another root at $x = -1$ and a double root. Fully factorise $P(x)$. **1**

iii) Sketch $y = P(x)$. **2**

c) A tangent to the parabola $x^2 = 4ay$ at the point T $(2at, at^2)$ meets the parabola $x^2 = -4ay$ in two points P and Q. Show that the locus of M, the midpoint of PQ, is also a parabola and give its equation. **3**

End of Question 7
End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$