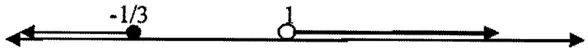
Western Region

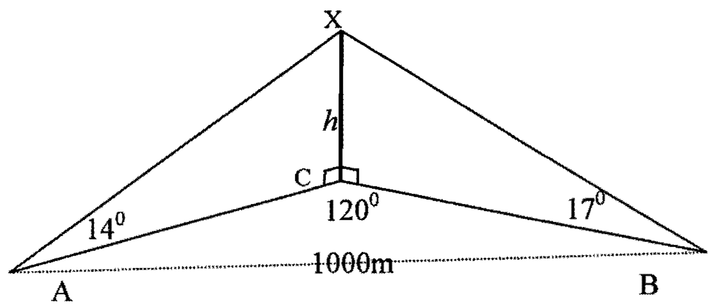
2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Solutions

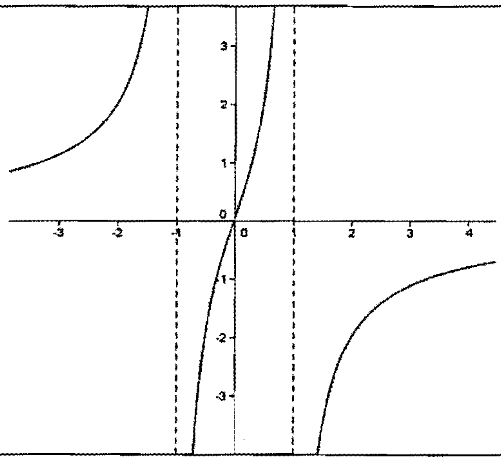
Solutions Question 1 2009	Marks/Comments
a. $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)=\left(\frac{2\times12+3\times-3}{5},\frac{2\times-4+3\times6}{5}\right)$ $=(3,2)$	1 1
b. $\sin(60+45)=\sin60\cos45+\cos60\sin45$ $=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$	1 1
c. $x\neq1$, $4=3-3x$, $x=-\frac{1}{3}$ are critical points test x = -1TRUE 4/2$<$3 test x = 0 FALSE 4/1 $>$ 3 test x = 2 TRUE 4/-1$<$3 solution is $x\leq-\frac{1}{3}$ *or* $x>1$  	1 **Or by multiplying by (1-x)^2** **1 pay 2 for any legitimate** **1 must have open circle on x = 1**
d. $u=\cos x$ $\frac{du}{dx}=-\sin x$ $du=-\sin x\,dx$ $I=-\int u^2du=-\frac{1}{3}u^3=-\frac{1}{3}\cos^3x+c$	**1** **1 ignore constant of integration**
e. $L=\lim_{x\to\infty}\frac{\frac{3x^3}{x^2}-\frac{2x}{x^2}}{1+\frac{4}{x^2}}=\lim_{x\to\infty}\frac{3x-\frac{2x}{x^2}}{1+\frac{4}{x^2}}=\lim_{x\to\infty}3x=\infty$	**Or divide by x^3** **1**
f $m_2=\frac{1}{\sqrt{3}}$, $m_1=1$ $\tan\theta=\left\|\frac{m_2-m_1}{1+m_2m_1}\right\|$ $=\left\|\frac{\frac{1}{\sqrt{3}}-1}{1+\frac{1}{\sqrt{3}}}\right\|$ $=\left\|\frac{1-\sqrt{3}}{\sqrt{3}+1}\right\|$ $\theta=15^o$	**1** **1** **/12** **pay 1 for successful sub in formula 2 if correct conclusion**

Solutions Question 2 2009	Marks/Comments
a.(i) x^3-4x^2+7x-6 $P(2)=0 \quad 2^3-4\times 2^2+14-6=0$ $\therefore (x-2)$ is a factor (ii)Dividing $\dfrac{x^3-4x^2+7x-6}{x-2}=x^2-2x+3$ a quadratic with negative discriminant and no real roots	1 1 1
b (i) $\dfrac{1-\cos 2x}{\sin 2x}=\dfrac{1-(\cos^2 x-\sin^2 x)}{2\cos x\sin x}=\dfrac{1-(1-\sin^2 x-\sin^2 x)}{2\cos x\sin x}$ $=\dfrac{2\sin^2 x}{2\cos x\sin x}=\dfrac{\sin x}{\cos x}=\tan x$ (ii) $\dfrac{1-\cos 30^\circ}{\sin 30^\circ}=\dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\dfrac{2-\sqrt{3}}{1}=2-\sqrt{3}$	 1 1
c. $\left[\dfrac{1}{2}\tan^{-1}\dfrac{x}{2}\right]_{2/\sqrt{3}}^{2\sqrt{3}}=\dfrac{1}{2}\left(\tan^{-1}\sqrt{3}-\tan^{-1}\dfrac{1}{\sqrt{3}}\right)=\dfrac{1}{2}\left(\dfrac{\pi}{3}-\dfrac{\pi}{6}\right)=\dfrac{\pi}{12}$	3 **1 per step**
d. (i) $\dfrac{7!}{2!2!}=1260$ (ii) $\dfrac{5!}{2!}=60$ (iii) Fix the E or any of the single letters $\dfrac{6!}{2!2!}=180$	1 1 2 **/12**

Solutions Question 3 2009	Marks/Comments
a. $\int_2^3 \frac{xdx}{x^2-2} = \left[\frac{1}{2}\ln(x^2-2)\right]_2^3 = \left[\ln\sqrt{x^2-2}\right]_2^3 = \ln\sqrt{7}-\ln\sqrt{2}$ $= \ln\left(\frac{\sqrt{7}}{\sqrt{2}}\right)$	**3 lose 1 per error** **Any equivalent exact value is okay.**
b. $-1 \le \frac{3x}{2} \le 1 \qquad -\frac{2}{3} \le x \le \frac{2}{3}$ domain range $0 \le y \le \pi$	**1** **1**
c $\frac{7}{2}\sin\theta + 2\cos\theta = 4 \quad \frac{7t}{1+t^2} + 2\frac{1-t^2}{1+t^2} = 4$ $7t + 2 - 2t^2 = 4 + 4t^2 \quad 6t^2 - 7t + 2 = 0 \quad (3t-2)(2t-1) = 0$ $\tan\frac{\theta}{2} = \frac{1}{2}, \frac{2}{3} \quad \frac{\theta}{2} = \tan^{-1}\left(\frac{1}{2}\right)$ *and* $\tan^{-1}\left(\frac{2}{3}\right)$ $\theta = 53°8'$ *or* $67°23'$	**1** **1** **1**
d 	**1**
d (ii) $AC = b = h\cot 14 \quad BC = a = h\cot 17$ $c^2 = a^2 + b^2 - 2ab\cos C$ $1000^2 = h^2\cot^2 14 + h^2\cot^2 17 - 2\times h\cot 14 h\cot 17\cos 120$ $= h^2(\cot^2 14 + \cot^2 17 + \cot 14\cot 17)$ $= h^2 \times 39.9035$ $h = \sqrt{1000000 \div 39.9035} = 158.3 \approx 158m$	**1** **1** **1** **/12**

Solutions Question 4 2009	Marks/Comments
a $x^3-5x^2+7x+5=0$	
(i) $\alpha+\beta+\gamma=-\frac{b}{a}=5$	1
(ii) $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=7$	1
(iii) $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\gamma\alpha$	1
$\therefore \alpha^2+\beta^2+\gamma^2=5^2-2\times 7=11$	1
b (i) $\sqrt{3}\sin 2\theta-\cos 2\theta=2\left(\frac{\sqrt{3}}{2}\sin(2\theta)-\frac{1}{2}\cos(2\theta)\right)$	**1 for r and α**
$=2\sin\left(2\theta-\frac{\pi}{6}\right)$	**1 for correct form**
noting $\sin(A-B)=\sin A\cos B-\cos A\sin B$	
(ii) $2\sin\left(2\theta-\frac{\pi}{6}\right)=1$ $\left(2\theta-\frac{\pi}{6}\right)=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6},\frac{5\pi}{6}$	**1 for solving for 2θ**
$2\theta=\frac{\pi}{3},\ \pi$ $\theta=\frac{\pi}{6},\frac{\pi}{2}$	**1 for θ**
c) i) $T=D+Ce^{-kt}$ $\frac{dT}{dt}=-k(Ce^{-kt}+D-D)=-k(T-D)$	1
ii)	
$D=25, C=1325$ $T(10)=720=25+1325e^{-10k}$	**1 for equation**
$\frac{695}{1325}=e^{-10k}$ $\ln\frac{695}{1325}\div -10=k=0.0645255...$	**1 for *k***
$50=25+1325e^{-0.0645255t}$	**1 for answer.**
$e^{-0.0645255t}=\frac{25}{1325}$	**/12**
$t=\ln\frac{25}{1325}\div 0.0645255=61.53\text{min}$	

Solutions Question 5 2009	Marks/Comments
a. $f'(x) = 4x^3 - 15x^2 + 22x - 12 \quad f'(1.3) = 0.038$ $f''(x) = 3.28$ $x_2 = x_1 - \dfrac{f'(x_1)}{f''(x_1)} = 1.3 - \dfrac{0.038}{3.28} = 1.288...$ $f'(1.288) = -0.00128$ a ii $f(1.3) = 0.8611$ x=1.3 is so close to the minimum that the curve cannot get much lower.	1 1 1 1
b) 0.5 chance not even $0.5^7 = .0078125$ chance of not throwing < 1% Therefore chance of at least 1 even > 99% if 7 rolls	1 1
c $\dfrac{ds}{dt} = \dfrac{ds}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{3s^2} \times 200 = \dfrac{200}{675} = \dfrac{8}{27}$ $\dfrac{dSA}{dt} = \dfrac{dSA}{ds} \times \dfrac{ds}{dt} = 12s \times \dfrac{8}{27} = 53\frac{1}{3} cm^2 s^{-1}$	1 1 1
d) If $n = 1$ $1^3 + 2^3 + 3^3 = 36 = 9 \times 4$ so the result holds when $n = 1$ Assume that when $n = k$ $(k)^3 + (k+1)^3 + (k+2)^3 = 9m,\ m \in C$ RTP $(k+1)^3 + (k+2)^3 + (k+3)^3 = 9p,\ p \in C$ $LHS = 9m - k^3 + (k+3)^3 = 9m - k^3 + k^3 + 9k^2 + 27k + 27$ $= 9(m + k^2 + 3k + 3) = 9p$ as req[d] Hence since true for $n = 1$, and since if true for $n = k$. Also true for $n = k + 1$, by induction the result holds for all positive integers n	1 1 1 **/12** **Penalise 1 if conclusion not stated**

Solutions Question 6 2009	Marks/Comments
a) i) $x \neq \pm 1$	**1**
a) ii) $h(2) = -2, \;\; h(-2) = 2$	**1**
a) iii) $h(x) = \dfrac{3x}{1-x^2}$ $\quad \dfrac{vu'-uv'}{v^2} = \dfrac{3(1-x^2)-3x(-2x)}{(1-x^2)^2} = \dfrac{3+3x^2}{(1-x^2)^2}$ positive numerator and denominator $\therefore$ never zero	**1**
a) iv) 	**2 award 1 for progress**
b) $V = \pi \int_0^{\frac{\pi}{3}} y^2 dx = \pi \int_0^{\frac{\pi}{3}} \sec^2 dx = \pi [\tan x]_0^{\frac{\pi}{3}} = \sqrt{3}\pi \; units^3$	**2**
c) i) $\dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt} = v.\dfrac{dv}{dx} = \dfrac{d\left(\frac{1}{2}v^2\right)}{dv}\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{v^2}{2}\right)$	**1**
c) ii) α $\quad v^2 = 36 - 4x^2 \quad \ddot{x} = \dfrac{d}{dx}\left(18 - 2x^2\right) = -4x$ $\ddot{x} = -n^2 x$ with $n = 2$ signifying SHM	**1**
c) ii) β $\quad$ At the endpoints $v = 0, \;\; x = \pm 3$ amplitude = 3 $T = \dfrac{2\pi}{n} = \pi$	**1** **1**
c) ii) γ $\quad x = 3\sin(2t)$ OR $x = 3\cos(2t - \frac{\pi}{2})$	**1** **/12**

Solutions Question 7 2009	Marks/Comments
a) $T\hat{O}P = B\hat{O}P$ (given bisector) $O\hat{T}P = T\hat{A}B$ (angle between chord and tangent equals the angle in the alternate segment) $O\hat{Q}A = O\hat{P}T$ (angle sum triangle) But $O\hat{P}T = P\hat{T}Q + P\hat{Q}T$ and $O\hat{Q}A = P\hat{T}Q + T\hat{P}Q$ (exterior angle triangle) $\therefore P\hat{Q}T = P\hat{T}Q$ (equals - $P\hat{T}Q$) ΔTPQ is isosceles as req$^{\underline{d}}$	1 1 1
If tan θ = $\frac{5}{12}$ $\cos\theta = \frac{12}{13}$ $\sin\theta = \frac{5}{13}$ $\ddot{x} = 0$ $\dot{x} = \int \ddot{x}dt = c$ $\ddot{y} = -10$ $\dot{y} = \int \ddot{y}dt = -10t + c$ $= -10t + 10$ b) $\dot{x} = 26 \times \frac{12}{13} = 24$ $y = \int \dot{y}dt = -5t^2 + 10t + c$ $x = \int \dot{x}dt = 24t + c = 24t$ $= -5t^2 + 10 + 15$ since components given in first line and origin is 15m below point of projection	1 1 **explanation of evaluation of Cs of I must be given**
b)ii) Impact when $t = 0$ $0 = -5t^2 + 10t1 = 15$ $0 = t^2 - 2t - 3 = (t-3)(t+1)$ $whence\, t = 3$ $x(3) = 24 \times 3 = 72m$	1 1
c) i) $PS = \sqrt{(2t-0)^2 + (t^2-1)^2} = \sqrt{t^4 + 2t^2 + 1} = t^2 + 1$ $PM = t^2 + 1$ which is the distance of P above the x axis plus the distance of the directrix below the x axis.. The lengths are equal Δ *PSM is isocseles*	1 1
c)ii) $y = \frac{x^2}{4}$ $y' = \frac{x}{2} = t\ atP$ $m_{SM} = \frac{-2}{2t} = -\frac{1}{t}$ ∴ The two lines are perpendicular	1 1
c)iii) The altitude of an isosceles triangle bisects the angle at the apex AND $A\hat{P}B = C\hat{P}M$ vertically opposite ∴ $A\hat{P}B = S\hat{P}C$ as req$^{\underline{d}}$	1 **/12**