2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

Question 1 (12 Marks) Begin a new booklet. **Marks**

(a) Divide the interval from A (-3, 6) to B (12, -4) in the ratio 2:3 **2**

(b) Find the value of $\sin 105^\circ$ in simplest exact form **2**

(c) Solve the inequality $\dfrac{4}{1-x} \le 3$ and graph your solution on the number line **3**

(d) Use the substitution $u = \cos x$, to find $\int \cos^2 x \sin x dx$ **2**

(e) Find $\displaystyle\lim_{x \to \infty} \frac{3x^3 - 2x}{x^2 + 4}$ **1**

(f) Find the acute angle between the lines: $x - \sqrt{3}y + 1 = 0$ **2**

$y = x - 4$

End of Question 1

Question 2 (12 Marks) Begin a new booklet. **Marks**

(a) i) Show that $x-2$ is a factor of x^3-4x^2+7x-6 **1**

ii) Show why $x^3-4x^2+7x-6=0$ has only 1 real root. **2**

(b) (i) Prove $\dfrac{1-\cos 2x}{\sin 2x}=\tan x$ **1**

(ii) Hence express $\tan 15^\circ$ in simplest exact form. **1**

(c) Find the value of $\displaystyle\int_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}} \frac{dx}{x^2+4}$ **3**

(d) How many distinct permutations of the letters of the word **ARRANGE** are possible

(i) In a straight line **1**

(ii) In a straight line when the "word" begins and ends with the letter R. **1**

(ii) In a circle **2**

End of Question 2

Question 3 (12 Marks) Begin a new booklet. **Marks**

(a) Determine the exact value of $\int_2^3 \frac{x\,dx}{x^2-2}$ **3**

(b) State the domain and range of $y = \cos^{-1}\left(\frac{3x}{2}\right)$ **2**

(c) If we take $t = \tan\frac{\theta}{2}$ then $\tan\theta = \frac{2t}{1-t^2}$ **3**

Use the t results or otherwise to obtain θ correct to the nearest minute

when $\frac{7\sin\theta}{2} + 2\cos\theta = 4$

(d) A tower CX is observed at an angle of elevation 14° from a point A on level ground.

The same tower is observed from B, 1 km from A, with an angle of elevation 17°.

$A\hat{C}B = 120^\circ$. C is the base of the tower.

(i) Draw a diagram showing this information. **1**

(ii) Calculate h, the height of the tower CX. (nearest m) **3**

End of Question 3

Question 4 (12 Marks) Begin a new booklet. **Marks**

(a) The polynomial equation $x^3 - 5x^2 + 7x + 5 = 0$ has 3 roots, α, β, γ

(i) Find $\alpha + \beta + \gamma$ **1**

(ii) Find $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

(iii) Find $\alpha^2 + \beta^2 + \gamma^2$ **2**

(b) (i) Express $\sqrt{3}\sin 2\theta - \cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, α acute. **2**

(ii) Hence solve $\sqrt{3}\sin 2\theta - \cos 2\theta = 1$; $0 \le \theta \le \pi$. Answer in exact form. **2**

(c) Newton's Law of Cooling states that the rate of change of temperature of a body is proportional to the difference between the temperature of the body and its surrounds.

$$\frac{dT}{dt} = -k(T - D) \quad \text{where } D \text{ is the surrounding temperature}$$

(i) Show that $T = D + Ce^{-kt}$ satisfies Newton's Law. **1**

(ii) An ingot of Aluminium has an initial temperature of $1350^\circ C$ **3**

After 10 minutes in an environment at $25^\circ C$ its temperature has fallen to $720^\circ C$. What total time elapses for the ingot to cool to $50^\circ C$

End of Question 4

Question 5 (12 Marks) Begin a new booklet. **Marks**

(a) The function $f(x) = x^4 - 5x^3 + 11x^2 - 12x + 6$

This function has only 1 minimum near $x = 1.3$

(i) Use one application of Newton's Method to obtain a better approximation to the x value **of this minimum**. **3**

(ii) Justify why $f(x) = 0$ has no roots **1**

(b) How many times should a die be thrown so that the probability of throwing an even number is greater than 0.99? **2**

(c) A cube, side s, is growing at a rate of 200cm^3 per second. **3**

At what rate is the surface area growing at the moment when $s = 15\text{cm}$?

(d) Prove by Mathematical Induction that, **3**

$(n)^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for all positive whole numbers n

End of Question 5

Question 6 (12 Marks) Begin a new booklet. **Marks**

(a) Consider the function $h(x) = \dfrac{3x}{1-x^2}$ for which $\lim\limits_{x \to \infty} \dfrac{3x}{1-x^2} = 0$

(i) Describe the domain of $h(x)$ **1**

(ii) Find $h(-2)$ and $h(2)$ **1**

(iii) Show why $h(x)$ has no turning points **1**

(iv) Sketch $h(x)$ showing the important features **2**

(b) Find the volume generated when $y = \sec x$ **2**

between $x = 0$ and $x = \dfrac{\pi}{3}$ is rotated around the x axis.

Express your answer in simplest exact form.

(c) (i) Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\dfrac{v^2}{2}\right)$ **1**

(ii) A particle is moving in a straight line with $v^2 = 36 - 4x^2$

α Prove the particle is undergoing SHM **1**

β What is the amplitude and period of the motion? **2**

γ If the particle is initially at the origin, write an expression for its displacement in terms of t **1**

End of Question 6

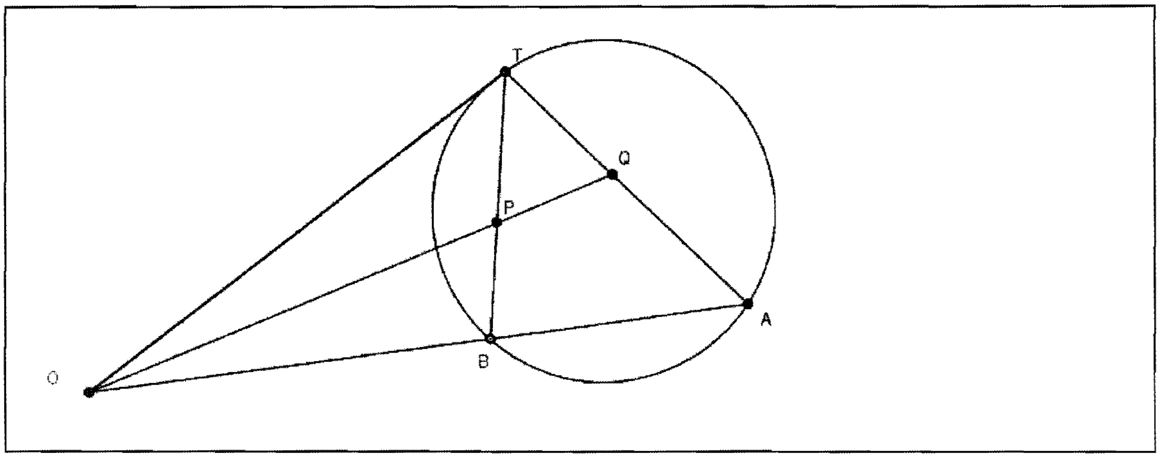
Question 7 (12 Marks) Begin a new booklet. **Marks**

(a) AB is a chord in a circle. AB is produced to O outside the circle. **3**

From O. the tangent OT is drawn to the circle.

The bisector of $T\hat{O}B$ Meets TB at P and TA at Q.

Prove ΔTPQ is isosceles



(b) A rock is hurled from the top of a 15m cliff with an initial velocity of 26ms^{-1} at an angle of projection equal to $\tan^{-1}\left(\frac{5}{12}\right)$ above the horizontal.

The cliff overlooks a flat paddock.

The equations of motion of the stone are $\ddot{x} = 0$ and $\ddot{y} = -10$

(i) Taking the origin as the base of the cliff, show the components of the rock's displacement are, $x = 24t$ and $y = -5t^2 + 10t + 15$ **2**

(ii) Calculate the time until impact with the paddock, and the distance of the impact from the base of the cliff. **2**

Question 7 continues on page 9

Question 7 (12 Marks) continued **Marks**

(c) The parabola $x^2 = 4y$ is shown in the diagram.

The point P has coordinates $(2t, t^2)$. S(0, 1) is the focus and

M is the foot of the perpendicular from P on the directrix. MP is produced to A.

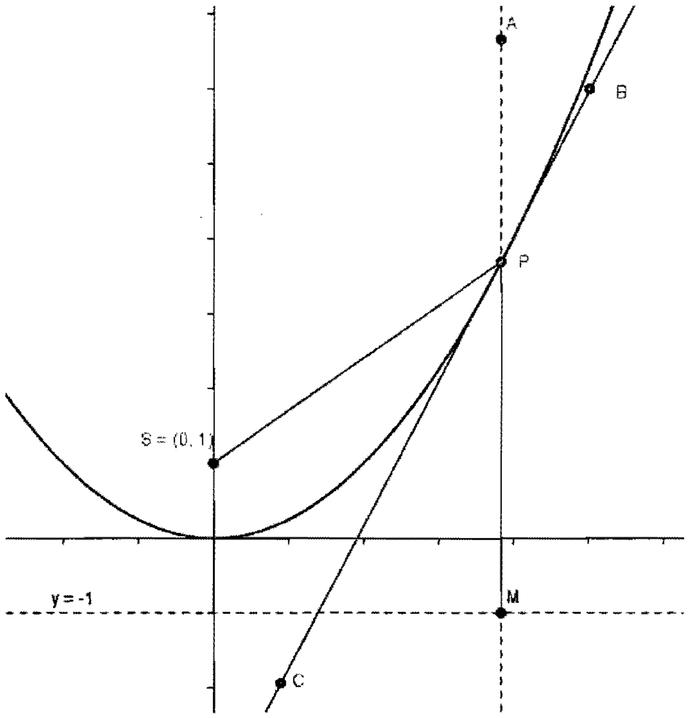
BPC is tangent to the parabola at P.

(i) Find the length PS and PM. Describe ΔPSM **2**

(ii) Find the gradient of the tangent BPC and of SM. **2**

What is true about the tangent and SM?

(iii) Prove $A\hat{P}B = S\hat{P}C$ **1**



End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$