WR 2003

QUESTION 7 (12 MARKS) Use a SEPARATE Sheet of Paper **Marks**

(a) The population of a species of bacteria grows such that the population P (bacteria) at a time t (minutes) is given by : $P = 2000e^{kt}$

(i) Show that the rate of increase of the population is proportional to the size of the population at that time. **1**

(ii) Given that the initial population doubles after 4 minutes calculate the value of k, correct to 3 significant figures. **2**

(iii) Find the population after 6 minutes. (correct to the nearest whole number) **1**

(b) On separate diagrams, draw the graphs of the following:

(i) $x^2 + y^2 \leq 4$ **1**

(ii) $y \geq 1 - \cos 2x$. for $-\pi \leq x \leq \pi$. **2**

(iii) The intersection of the two inequalities above. **2**

(c) Find the values of P, Q and R if $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$ **3**

WR 2004

Question 7 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) A married couple borrow \$250 000 from a bank to be repaid in equal monthly installments of \$M at the end of every month. Interest at the rate of 6.24 % p.a. is charged on the amount owing for that month.

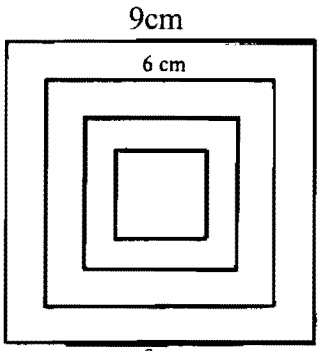
If A_n is the amount owing at the end of the nth month:

i. Write down an expression for A_1. **1**

ii. Show that $A_3 = 250\,000(1.0052)^3 - M(1.0052^2 + 1.0052 + 1)$ **2**

i. Calculate the value of each monthly installment (\$M) if the loan is to be repaid in 25 years. **3**

(b) The diagram below shows the beginning of an infinite set of squares. The outer square has a side of length 9 cm and each successive square has a side of length $\frac{2}{3}$ of that of the previous square. Find the sum of the perimeters of all the squares. **3**



c) Use Simpson's Rule with five function values to find the approximate value of $\int_1^5 (x^2+1)dx$ to three decimal places. **3**

WR 2005

Question 7 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) A population of bacteria in a medium are growing at a rate proportional to the current population. The population obeys the model $P = P_0 e^{kt}$, where P_0 is the population of bacteria at noon on 1 August and t is measured in hours. When $t = 6$ the population has grown from 900 000 to 1.4 million.

(i) Show that $\frac{dP}{dt} = kP$ **1**

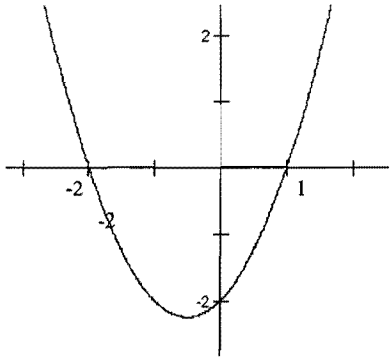
(ii) What is the value of k? **2**

(iii) What will the population be when $t = 10$? **1**

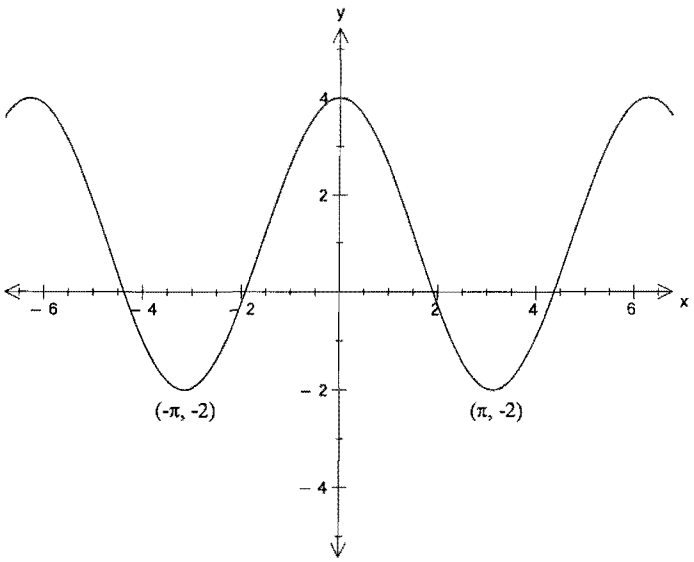
(iv) When will the population reach 3 million? **1**

(b) Write the functions represented by the following graphs

(i) **1**

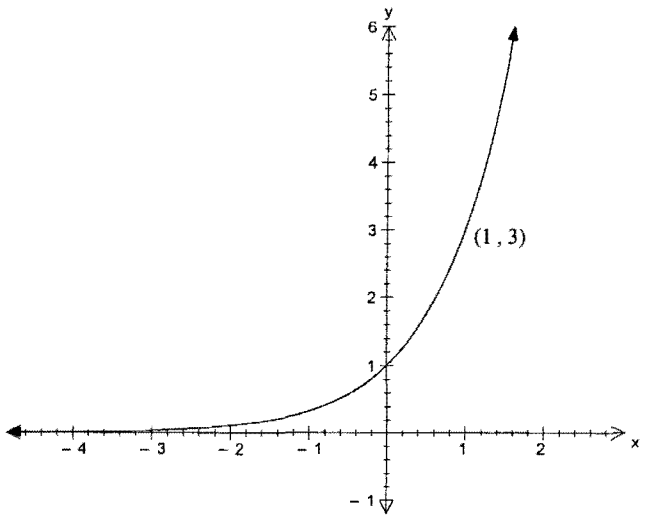


(ii) **2**

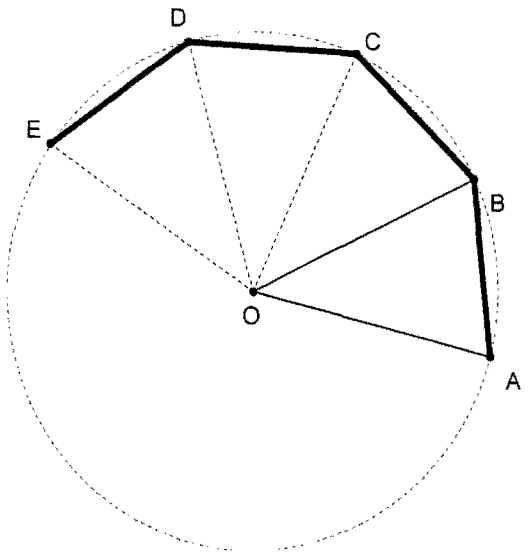


Marks

(iii) **1**



(c) ABCDE..... is a regular polygon with n sides inscribed within a unit circle with centre O.



(i) Explain why $A\hat{O}B = \dfrac{2\pi}{n}$ **1**

(ii) Write an expression for the area of the polygon ABCDE... in terms of n. **1**

(iii) Show $\lim\limits_{n \to \infty} \left(\dfrac{n}{2} \sin\left(\dfrac{2\pi}{n} \right) \right) = \pi$ **1**

WR 2006

Question 7 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) For the curve $y = x^3 - 3x^2 - 9x + 4$, Find:

i. the stationary points and determine their nature. **3**

ii. the point of inflection. **1**

iii. Sketch the curve for the domain $-2 \le x \le 4$. **2**

(b) The graph of a function has the following properties:

- Passes through the origin and has minimum turning point at (4,0)
- Concave up for $x > 3$ and $x < 0$
- Increasing for $-2 < x < 2$ and $x > 4$.

Sketch a possible graph of the function. **2**

(c) For what values of k does the equation $x^2 + (k+2)x + (k+2) = 0$ have equal roots. **2**

(d) The limiting sum of a series $3 + x + x^2 + \ldots\ldots$ is 18. **2**
If $|x| < 1$, find the value of x.

WR 2007

Question 7 (12 marks) Begin a SEPARATE sheet of paper

(a) Let A be the point (-2, 0) and B be the point (6, 0).

At P (x, y), PA meets PB at right angles.

(i) Show that the gradient of PA is $m_1 = \dfrac{y}{x+2}$ **1**

(ii) Find an equation for the locus of P **2**

(b) The velocity of an object is given by the equation $v = 6t - 8 - t^2$

Where time (t) is in seconds and velocity (v) in metres/second

It begins its motion at $x = 5$ metres.

(i) Find an equation for the displacement of the object **2**

(ii) At what 2 times is the object stationary? **1**

(iii) Find the distance travelled by the object between $t = 3$ and $t = 5$ **2**

(iv) What is the maximum velocity of the object? **1**

(c) Two dice are biased so that, the probability of a six is $\frac{3}{8}$ and of each other number is $\frac{1}{8}$.

Find the probability of

(i) Rolling a double six **1**

(ii) Rolling the two dice so that neither is a six **1**

(iii) Only 1 six appears when the two dice are rolled **1**

WR 2008

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) For the parabola with equation $x^2 = -8y$.

i) Find the coordinates of the focus (S) of the parabola. **1**

ii) Find the equation of the directrix of the parabola. **1**

iii) Show that the point A(-8, -8) lies on the parabola. **1**

iv) Find the equation of the focal chord of the parabola which passes through A. **2**

v) Find the equation of the tangent to the parabola at A. **2**

b) i) Show that the curves $y = x^2 - 3x$ and $y = 5x - x^2$ intersect at the points (0, 0) and (4, 4). **2**

ii) Find the area enclosed between the two curves. **3**

WR 2009

Question 7 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) The parabola $y = x^2$ and the line $y = x + 2$ intersect at points A and B respectively. Find the coordinates of the points A and B. Hence find the area bounded by the parabola and the line. **4**

(b) The minute hand on a clock face is 12 centimetres long.
In 40 minutes

i. Through what angle does the hand move (in radians)? **1**

ii. How far does the tip of the hand move? **1**

iii. What area does the hand sweep through in this time? **1**

(c) Use Simpson's rule to evaluate $\int_1^{2.5} f(x)\,dx$, to 1 decimal place **2**
using the 7 function values in the table below.

x	1·00	1·25	1·50	1·75	2·00	2·25	2·50
$f(x)$	3·43	2·17	0·38	1·87	2·65	2·31	1·97

(d) A function is defined by the following features: **3**

$\frac{d^2y}{dx^2} > 0$ for $x < -1$ and $1 < x < 3$.

$\frac{dy}{dx} = 0$ when $x = -3, 1$ and 5.

$y = 0$ when $x = 1$.

Sketch a possible graph of the function.