QUESTION 8   (12 MARKS)   Use a SEPARATE Sheet of Paper

(a)   Over the 20 days of a conference, the profit at the cafeteria is found to drop by a constant amount from each day to the next, becoming a loss after a certain amount of time. On the 2nd day there was a profit of \$700 and on the 6th day a loss of \$500.

        (i)   Find the Profit on the first day and the amount by which the profit dropped from each day to the next.   **2**

        (ii)   Find the loss on the 20th day.   **1**

        (iii)   Find the total profit (or loss) over the 20 days of the conference..   **1**

(b)   A parabola has equation $x - 2 = \dfrac{-y^2}{12}$

        (i)   Sketch the parabola showing it's vertex, focus and directrix.   **3**

        (ii)   Give the length of the focal chord which is perpendicular to the axis (i.e. the latus rectum)   **1**

(c)   (i)   Sketch the curves $y = \sin x$ and $y = \cos x$ on the same set of axes over the domain $0 \le x \le \pi$.   **1**

        (ii)   Use your graph to show that the equation $\sin x = \cos x$ has only one solution, $x = \dfrac{\pi}{4}$, over the domain $0 \le x \le \pi$..   **1**

        (iii)   Show that, at the point of intersection of the curves, the product of the gradients of the tangents to the two curves is $-\dfrac{1}{2}$.   **2**

(a)   For the Parabola $x^2 - 6x - 2y + 7 = 0$, find:

        i.   The focal length   **2**

        ii.   The coordinates of the Vertex   **1**

        iii.   The coordinates of the focus   **1**

        iv.   The equation of the directrix   **1**

        v.   The length of the latus rectum.   **1**

(b)   The area below the curve $y = e^{-x}$, between the values $x = 0$ and $x = 4$ is   **3**
rotated around the x – axis. Calculate the volume of the solid of revolution.

(c)   The graph of $y = f(x)$ is drawn below.
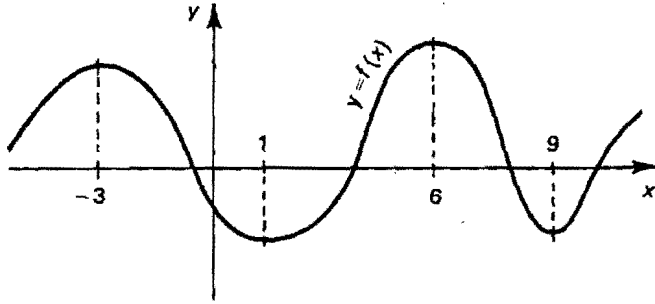
When is the derivative of $f(x)$ i.   less than zero   **1**
                                      ii.   equal to zero   **1**

    iii. Draw a possible sketch of $y = f'(x)$ between the points $x = 1$ and $x = 9$ **1**

Question 8   **(12 Marks)**   Use a Separate Sheet of paper   **Marks**

(a)   (i)   Show that $y = mx - 2m^2$ is tangent to the parabola $x^2 = 8y$ **2**

        (ii)   Find the two values of m for which the tangent passes through (2, -4)   **2**

(b)   (i)   Use Simpson's rule with 5 function values to evaluate   **3**

$$\int_0^4 \frac{\sqrt{144 - 9x^2}}{4} dx$$

        ii)   The formula $A = \frac{\pi ab}{4}$ where $a=4$ and $b = 3$, gives the exact   **1**

                value of the integral above. Comment on the accuracy of your answer
                from (i) compared to the exact answer.

        Consider the parabola $2y = x^2 - 4x$.

        i)   Rewrite it in the form $4a(y - k) = (x - h)^2$ **2**

        ii)   Give the coordinates of the focus.   **1**

        iii)   Give the equation of the directrix.   **1**

Question 8   (12 Marks)   Use a Separate Sheet of paper   **Marks**

(a)   Find, in exact form, the volume of the solid of revolution formed   **3**
       when the area bounded by the curve $y = \log_e 3x$, the y-axis, from
       $y = 1$ to $y = 3$ is rotated about the y-axis.

(b)   The acceleration of a particle travelling in a straight line is given by

$$\frac{d^2s}{dt^2} = 8 - 6t \ .$$

       The particle is initially at the origin and travelling at 5 m/s to the right.

       i.   Find equations for the velocity and displacement of the particle.   **2**

       ii.   At what time does the particle return to the origin?   **2**
               Find the velocity of the particle at that time.

(c)   Solve $2\log_a x - \log_a 4 = 2\log_a 8$ **3**

(d)   A woman buys a ticket in a raffle in which there are three prizes and 50
       tickets are sold. What is the probability that she:

       i.   does not win a prize?   **1**

       ii.   wins the third prize?   **1**

Question 8 (12 marks) Begin a SEPARATE sheet of paper   **Marks**

(a)   The graph of $y = f'(x)$ is shown.   The roots of $f'(x)$ are $x = $ -2, 0.5, and 3

C has x coordinate -0·95 and B has x coordinate 1·95
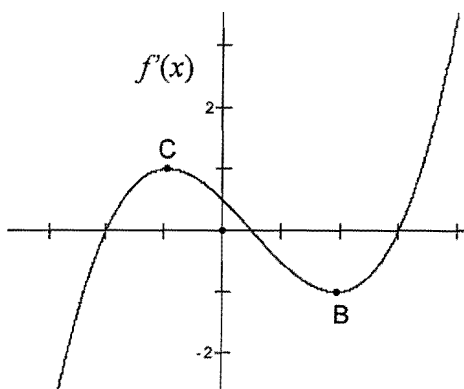
(i)   For what values of x is $f(x)$ 1

increasing?

(ii)   C is a local maximum on $f'(x)$.   2

What type of point occurs on $f(x)$

at the same x value as that shown at C.

Justify your answer.

(iii)   For what values of x is $f(x)$ concave down?   1

(b)   The curve $y = \log_e x$ between $x = e$ and $x = 3e$ is rotated around the x axis.

(i)   Write the integral which gives the value of this volume.   2

(ii)   Complete the table for this function write your answer to 2 decimal places   2

x	e	$2e$	$3e$
$\pi \times (f(x))^2$			

(iii)   Use Simpson's Rule with 3 function values to approximate the volume.   2

(c)   What is the domain and range for $y = \sqrt{9 - x^2}$ 2

a)   A city has a population which is growing at a rate that is proportional to the current
        population. The population at time t years is given by
        $$P = Ae^{kt}$$

     i)   Show that $P = Ae^{kt}$ satisfies the equation $\dfrac{dP}{dt} = kP$.   1

     ii)   If the population at the start of 2006 when $t = 1$ was 147 200 and at the start of   2
           2007 when $t = 2$ was 154 800, find the values of A and k.

     iii)   Find the population at the start of 2009.   1

     iv)   Find during which year the population will first exceed 200 000.   1

b)   In the diagram below, P is the midpoint of the side AB of the $\triangle$ ABC.
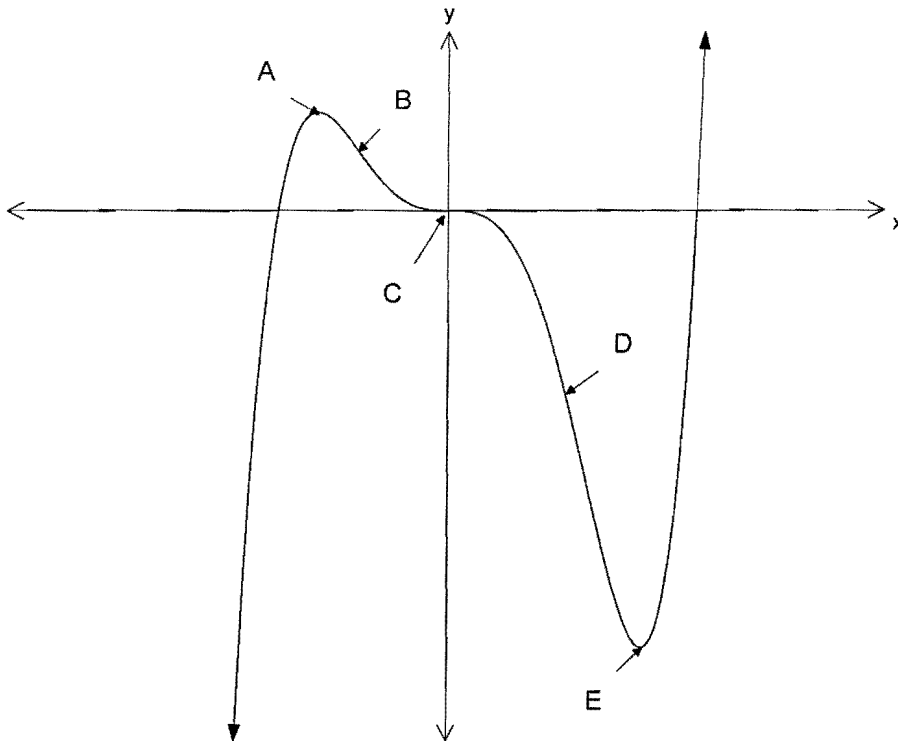   PQ is drawn parallel to BC.
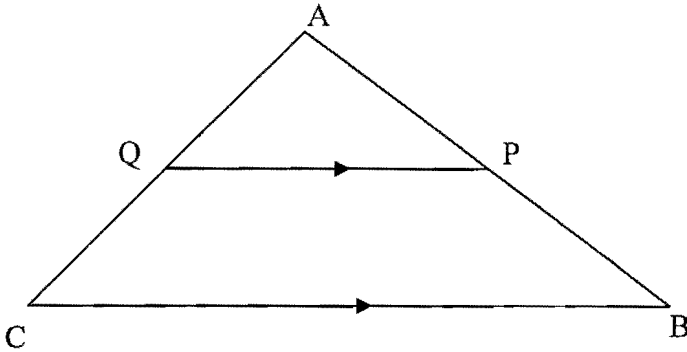
     i)   Prove that $\triangle$ ABC $\||$ $\triangle$ APQ.   2

     ii)   Explain why Q is the midpoint of AC.   2

c)   Find an approximation for $\displaystyle\int_{1}^{3} g(x)\,dx$ by using Simpson's Rule with the values in the   2

     table below.

x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

d)   Evaluate $\displaystyle\sum_{n=2}^{5} n^2 - 1$ 1

(a)   The graph of the curve *y = f(x)* is drawn below.

i.   Name the points of inflexion.   1

ii.   When is the graph decreasing?   1

iii.   Sketch the gradient function.   1

(b)   Steve borrows $15 000 for a new car. He decides to repay the loan
plus interest at 6% pa compounded monthly. He repays the loan in monthly
installments of $P.

i.   Show that after three months the amount that Steve owes   2
is $[15226·13 − P(3·015025)].

ii.   After two years of repaying his loan, Steve still owes $10 000   3
on the loan. What was the monthly repayment?

(c)   Sketch the graph of the parabola $2x = y^2 - 8y + 4$, showing the vertex,   4
focus and the directrix.