

These are suggested answers only. Any reasonable solution should be accepted.

Solutions	Marks/Comments
1 a) i) $(4x+3)(4x-3)$ ii) $5a(a+2)$	1 1
b) $3\sqrt{5} + \sqrt{4}\sqrt{5} = 3\sqrt{5} + 2\sqrt{5}$ $5\sqrt{5} = \sqrt{25}\sqrt{5} = \sqrt{125}$	1 1
c) $x^2y + 3xy^2$	1
d) $\lim_{x \rightarrow \infty} \left(3 - \frac{4x}{x^2} + \frac{5}{x^2} \right)$ $= 3$	1 1
e) $x^2 - x - 1 = 0$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$	1 1
f) $x = 0 \cdot 4\dot{2}$ $100x = 42 \cdot 4\dot{2}$ $99x = 42 \quad x = \frac{42}{99} = \frac{14}{33}$	1 1
g) 1.61	1
	/12
2 a) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{6^2 + 3^2}$ $= \sqrt{45}$ $= 3\sqrt{5}$	1 1 simplification not reqd
b) $m = \frac{-3}{6} = \frac{-1}{2}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26^\circ 34' = 27^\circ$	1 1 ignore rounding error
c) subbing $x = -5, y = 5$ or $y - 2 = -\frac{1}{2}(x - 1)$ $-5 + 2 \times 5 - 5 = 0$ true $x + 2y - 5 = 0$ subbing $x = 1, y = 2$ $1 + 2 \times 2 - 5 = 0$ true	1 1

d) $d = \frac{ Ax_1 + By_1 + C }{\sqrt{A^2 + B^2}}$ $x_1 = 1, y_1 = 6$	1
$d = \frac{ 1+12-5 }{\sqrt{5}} = \frac{8}{\sqrt{5}}$	1
e) $A = \frac{bh}{2} = 3\sqrt{5} \times \frac{8}{\sqrt{5}} \div 2 = 12$	1
f) call BC = 4 the base and the perpendicular distance to A = 6, the height. $4 \times 6 \div 2 = 12$	1
g) (-5, 1), (-5, 9), (7, 3)	1 for first, 1 more for all 3 /12
3. a) i) note $\sqrt[4]{x^3} = x^{\frac{3}{4}}$	1
$\frac{d}{dx} \left(x^{\frac{3}{4}} \right) = \frac{3}{4} x^{\frac{-1}{4}}$	1
ii) $vu' + uv'$ $\cos x \ln x + \frac{\sin x}{x}$	1
iii) $\frac{vu' - uv'}{v^2}$ $\frac{e^x \cos x - e^x \sin x}{(e^x)^2} = \frac{\cos x - \sin x}{e^x}$	1
b) $V = \pi \int_a^b x^2 dy$ noting $x = 4 - y$ i.e. $x^2 = 16 - 8y + y^2$	1
$V = \pi \int_0^3 (16 - 8y + y^2) dy = \pi \left[16y - 4y^2 + \frac{y^3}{3} \right]_0^3$ $= \pi(48 - 36 + 9)$ $= 21\pi$	1
c) $g(x) = \int g'(x) dx = x^3 - 4x - x^{-1} + c$ but $g(x) = 4$ when $x = 1$ i.e. $4 = 1 - 4 - 1 + c$ so $c = 8$ and $g(x) = x^3 - 4x - \frac{1}{x} + 8$	1 1 1 /12
4 a) i) $4 \times 250 \times 30 = \$30\,000$	1
ii) $250 \times 1.02^{120} = \$2\,691.29$	1
iii) $250 \times (1.02^{120} + 1.02^{119} + 1.02^{118} + \dots + 1.02)$	1

$$250 \times \frac{a(r^n - 1)}{r - 1} = 250 \times \frac{1.02(1.02^{120} - 1)}{1.02 - 1}$$

$$250 \times 498.02328 = \$124\ 505.83$$

$$\text{iv) } \$124\ 505.82 - \$30\ 000 = \$94\ 505.83$$

b) $\theta = 1$ and $A = \frac{1}{2}r^2\theta$

$$A = 0.5 \times 5^2 \times 1 = 12.5\text{cm}^2$$

c) $f'(x) = 2e^x$ $f'(0) = 2$ slope of tangent

$$y - 2 = 2x \quad y = 2x + 2$$

d)

asymptote $x = 2$
x intercept 3

/12

5 a) i) $\hat{BAM} = \hat{CDM}$ alternate angles parallel lines

$\hat{CMD} = \hat{BMA}$ vertically opposite angles

$\Delta AMB \parallel\!\!\!|| \Delta DMC$ equiangular

ii) Ratio of areas = 6.25 ($= 2.5^2$)

total area = $10 + 62.5 = 72.5$

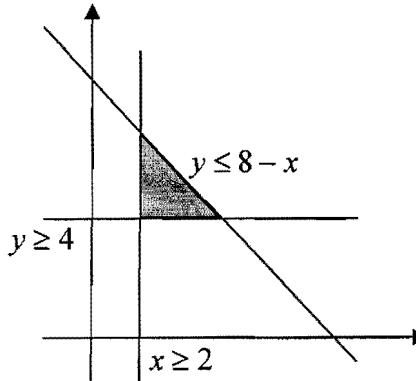
b) i) $I = \frac{1}{4}\sin(4x) + c$

ii) $\int_1^4 \frac{x}{x^2 + 4} dx = \frac{1}{2} \int_1^4 \frac{2x}{x^2 + 4} dx$

$$\frac{1}{2} [\ln(x^2 + 4)]_1^4 = \frac{1}{2} (\ln(e^8 + 4) - \ln 5)$$

$$\ln \sqrt{\frac{e^8 + 4}{5}}$$

c) Notes as to what graph is what should appear on the graph



1
1
1 for the subtraction
1

1

1
1

1
1

1
1
1

1

1 ignore c

1

1

1 for at least 1 area shown

1 for all 3 graphs shown

1 for correct area and annotation

5 d) $3 + 4 + 9 + 4 = 20$ $P = \frac{9}{20} \times \frac{8}{19} = \frac{18}{95}$	1 1 simplification not reqd /12
6 a) i) $x = 1, x = 2$ ii) point of inflection iii)	1 1 1 shape 1 intercept
iv) The graph of derivative indicates slope ≥ -2 between $x = 0$ and $x = 1$. The slope would have to be less than -3 To reach from $y = 3$ to $y = 0$ in the space of 1 unit	1 Ignore inaccuracy such as crossing the axis before first min.
b) i) $3 - \frac{x}{2} = \frac{x^2}{2} - 2x + 1$ $6 - x = x^2 - 4x + 2$ $0 = x^2 - 3x - 4$ $0 = (x - 4)(x + 1)$ $x = -1$ or 4	1 1 1
ii) $I = \int_1^4 3 - \frac{x}{2} - \left(\frac{x^2}{2} - 2x + 1 \right) dx$	1
$\int_1^4 2 + \frac{3x}{2} - \frac{x^2}{2} dx = \left[2x + \frac{3x^2}{4} - \frac{x^3}{6} \right]_1^4 = 8 + 12 - \frac{32}{3} - \left(\frac{3}{4} + \frac{1}{6} - 2 \right) = 10 \%$	1
c) (i) $P(WWW) = 0.4^3 = 0.064$ (ii) $P(LLL) = 0.6^3 = 0.216$ (iii) $P(\text{win at least 1 but not 3}) = 1 - (0.064 + 0.216)$ $= 1 - 0.28 = 0.72$	1 for 1/120 1 /12
7 a) i) $\frac{dP}{dt} = kP_0 e^{kt} = kP$ as required	1
ii) when $t = 6, P = 1400000, P_0 = 900000,$ $1400000 = 900000 e^{6k}$	1
$\frac{14}{9} = e^{6k}$	
$6k = \ln(14/9)$	1
$k = 0.073638792$	
iii) at $t = 10$ $P = 900000 \times e^{10 \times 0.073638792} = 1879541$	1

iv) $\frac{3000000}{900000} = e^{t \times 0.073638792}$ $\ln 3.3 = t \times 0.073638792 \quad t = 16\text{hrs}21'$	1
b) i) $y = (x+2)(x-1)$ ii) $y = 3 \cos x + 1$ iii) $y = 3^x$	1 1 for 1 element 1 for the others 1
c) i) $360^\circ = 2\pi$ therefore 2π by the number of sides n ii) $n \times \frac{1}{2}ab \sin C = n \times \frac{1}{2} \times 1 \times 1 \times \sin\left(\frac{2\pi}{n}\right) = \frac{n}{2} \sin\left(\frac{2\pi}{n}\right)$ iii) As n increases the area of polygon approaches area of the circle i.e. $\pi \times 1 \times 1 = \pi$	1 1 1 /12
8 a) i) Solving simultaneously $8mx - 16m^2 = x^2$ $x^2 - 8mx + 16m^2 = 0$ $(x - 4m)^2 = 0$ As this has only 1 answer the line is a tangent ii) $x = 2, y = -4$ and $-4 = 2m - 2m^2$ $2m^2 - 2m - 4 = 0 \quad 2(m-2)(m+1) = 0 \quad m = -1 \text{ or } 2$	1 1 1 1
b) i) $A \approx \frac{h}{3} (f(0) + f(4) + 2 \times f(2) + 4 \times (f(1) + f(3)))$ $\approx \frac{1}{3} \left(3 + 0 + 2 \times \frac{\sqrt{108}}{4} + 4 \times \left(\frac{\sqrt{135}}{4} + \frac{\sqrt{63}}{4} \right) \right)$ $= 9.2507855$ ii) $3 \times 4 \times \pi \div 4 = 9.424778$ the estimate slightly less than the true value	1 1 1 1 1 some effort to compare
c) i) $2y = x^2 - 4x \quad 2y + 4 = x^2 - 4x + 4$ $4 \times \frac{1}{2}(y+2) = (x-2)^2$ ii) Focus is $(2, -1.5)$ iii) Directrix $y = -2.5$	1 1 1
9 a) $n = 100, a = d = 5$ $\frac{n}{2}(2a + (n-1)d) = 50 \times (10 + 495) = 25250$	1 1 /12
b) $\frac{p}{\sin 72} = \frac{q}{\sin 39}$ so ... $\frac{p}{q} = \frac{\sin 72}{\sin 39} = 1.511$	1 + 1

c) i) $c^2 = a^2 + b^2 - 2ab \cos C$ = $= 200^2 + 105^2 - 2 \times 200 \times 105 \times \cos 150^\circ$ $= 87398$ $c = 295.63$	1	1 ignore rounding errors
ii) By Cos Rule $FH^2 = 200^2 + 205^2 - 2 \times 200 \times 205 \times \cos 13^\circ$ $FH = 45.06847$ $200^2 + (45.06847)^2 = 42031$ $205^2 = 42025$	1	
Square of hypotenuse \approx sum of squares on shorter sides. By Pythagoras Thm, angle is approx 90°	1	
Or use cos rule again to find angle, $\cos \theta = \frac{200^2 + FH^2 - 205^2}{2 \times 200 \times FH}$ $\theta = 89^\circ 58'$ approximately a right angle.	1 both equal radii 1 common side + conclusion	
d) (i) AC=BC radii of equal circles AE=BE radii of equal circles CE common $\therefore \Delta AEC \cong \Delta BEC$ (SSS) rule (ii) There are several possibilities a. CE bisects apex of isosceles ΔACB (corresponding angles in congruent triangles) and hence bisects base AB b. ACBE is rhombus, \therefore diagonals bisect at right angles. c. Prove $\Delta ACD \cong \Delta BCD$ (SAS), hence AD = DB Others possible.	2 marks for any reasonable explanation /12	
10 a) i) in this case the other side equals $200 - 2x$ Then $A = x(200 - 2x) = 200x - 2x^2$	1	
ii) $\frac{dA}{dx} = 200 - 4x$ which = 0 when $x = 50$ then $A = 100 \times 50 = 5000 \text{ m}^2$	1	
this is a maximum since if we take $w = 49$ then $l = 102$ and $A = 4998$	1	
b) i) $t = 0$ $v = 3 + 6 = 9$	1	
ii) $\int v \, dt = x = 3e^t - 6e^{-t} + c$ but $x = 0$ when $t = 0$ so $0 = 3 - 6 + c$ whence $c = 3$ then $x = 3e^t - 6e^{-t} + 3$	1	
iii) If $e^t = 3$, $e^{-t} = \frac{1}{3}$ and so $x = 3 \times 3 - 2 + 3 = 10$	1	
Under these conditions $3e^{2t} - 7e^t - 6 = 0$ $3 \times 9 - 7 \times 3 - 6 = 0$		
iv) $(3e^t + 2)(e^t - 3) = 0$ $t = \ln 3 = 1 \text{ minute } 6 \text{ seconds}$	1	
c) $a = 6$ $b = 3$	1	/12