Western Region

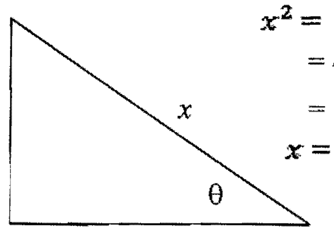
2009

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

Solutions

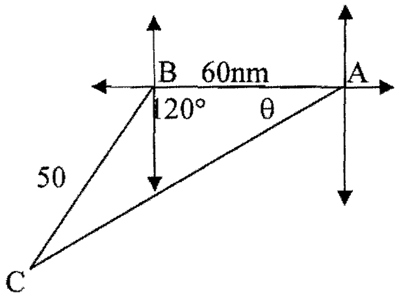
Solutions	Marks/Comments
Question 1	
(a) Let $x = 0{\cdot}53131..$ $10x = 5{\cdot}3131$ $100x = 53{\cdot}1313$ $1000x = 531{\cdot}3131$ $990x = 526$ $x = \frac{526}{990} = \frac{263}{495}$ $\therefore 3{\cdot}5\dot{3}\dot{1} = 3\frac{263}{495}$ or $3{\cdot}5\dot{3}\dot{1} = 5 + \frac{5}{10} + \frac{31}{1000} + \frac{31}{100000}....$ Limiting Sum $\frac{31}{1000} + \frac{31}{100000} + \frac{31}{10000000}....$ $a = \frac{31}{1000}$ $\quad S_\infty = \frac{a}{1-r}$ $r = \frac{1}{100}$ $\quad = \frac{\frac{31}{1000}}{1-\frac{1}{100}}$ $= \frac{31}{1000} \div \frac{99}{100}$ $= \frac{31}{990}$ $\therefore 3{\cdot}5\dot{3}\dot{1} = 3 + \frac{5}{10} + \frac{31}{990} = 3\frac{263}{495}$	**2 marks – 1 for correct method** **1 correct answer**
(b)  $x^2 = 7^2 + 8^2$ $= 49 + 64$ $= 113$ $x = \sqrt{113}$ Since $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$ 3rd Quadrant $\therefore \operatorname{cosec}\theta < 0$ $\therefore \operatorname{cosec}\theta = -\frac{\sqrt{113}}{7}$	**1 Mark – Correct Answer**
(c) $\frac{3{\cdot}24^2 - 2{\cdot}1^2}{\sqrt{36 + 2{\cdot}1}} = 0{\cdot}986242288$ $= 0{\cdot}986$	**1 Mark – Correct rounded answer**
(d) $\lvert 15 - 4x \rvert \le 3$ $15 - 4x \le 3$ or $15 - 4x \ge -3$ $-4x \le -12$ $\quad -4x \ge -18$ $x \ge 3$ or $\quad x \le 4\frac{1}{2}$	**2 Marks – 1 for each solution**
(e) $k = \frac{1}{3}m(v^2 - u^2)$ $724 = \frac{1}{3}m(14 \cdot 2^2 - 7.4^2)$	**2 Marks – 1 for substitution** **- 1 for answer**

$2172 = m(146 \cdot 88)$ $m = \frac{2172}{146 \cdot 88} = 14 \cdot 7875817$ $= 14 \cdot 8$ (3sf) (f) $3y = \sin\left(2x - \frac{\pi}{4}\right)$ $y = \frac{1}{3}\sin\left(2x - \frac{\pi}{4}\right)$ $\therefore$ amplitude $= \frac{1}{3}$ period $= \frac{2\pi}{2} = \pi$ (g) $130\% = \$67 \cdot 50$ $1\% = \frac{67 \cdot 50}{130} = 0 \cdot 51923\ldots\ldots$ Cost Price $= \frac{67 \cdot 50}{130} \times 100 = \$51 \cdot 92$	 **2 marks - 1 for period** **1 for amplitude** **2 Marks - 1 for 1%** **1 for Cost price**

Solutions	Marks/Comments
<u>Question 2</u> (a) $x - 2y + 9 = 0$ -----(1) $4x - y - 20 = 0$ -----(2) From (2) $y = 4x - 20$ ------(3) Sub (3) into (1) B is the point (7, 8) $x - 2(4x - 20) + 9 = 0$ $x - 8x + 40 + 9 = 0$ $-7x = -49$ $x = 7$ Hence $y = 8$	**2 marks – 1 for method 1 for correct answer**
(b) $m(AC) = \frac{4\frac{1}{2} - 0}{0 - 5}$ $= -\frac{9}{10}$ $y - y_1 = m(x - x_1)$ $y - 0 = -\frac{9}{10}(x - 5)$ $10y = -9x + 45$ $9x + 10y - 45 = 0$	**2 marks – 1 for gradient 1 for equation**
(c) $AC = \sqrt{(0 - 5)^2 + \left(4\frac{1}{2} - 0\right)^2}$ $= \sqrt{(-5)^2 + \left(4\frac{1}{2}\right)^2}$ $= \sqrt{25 + \frac{81}{4}}$ $= \frac{\sqrt{181}}{2}$	**2 marks – 1 for substitution 1 for answer**
(d) $m(BC) = \frac{8 - 0}{7 - 5} = \frac{8}{2} = 4$ $\therefore m(line) = -\frac{1}{4}$ $y - y_1 = m(x - x_1)$ $y - 4\frac{1}{2} = -\frac{1}{4}(x - 0)$ $4y - 18 = -x$ $x + 4y - 18 = 0$	**2 marks – 1 for gradient of line 1 for equation**
(e) $d = \frac{\lvert 9(7) + 10(8) - 45 \rvert}{\sqrt{9^2 + 10^2}} = \frac{\lvert 63 + 80 - 45 \rvert}{\sqrt{81 + 100}} = \frac{98}{\sqrt{181}}$ Area $= \frac{1}{2}bh = \frac{1}{2} \times \frac{\sqrt{181}}{2} \times \frac{98}{\sqrt{181}} = 24\frac{1}{2}$ square units.	**2 marks – 1 for substitution 1 for answer**
(f) $x - 2y + 9 \geq 0$ $4x - y - 20 \leq 0$ $9x + 10y - 45 \geq 0$	**2 marks - lose 1 mark for each incorrect**

Solutions	Marks/Comments
Question 3	
(a) i. $\frac{d}{dx}\left[3x\sqrt[3]{x}\right] = vu' + uv'$ $= 3 \times x^{\frac{1}{3}} + 3x \times \frac{1}{3}x^{-\frac{2}{3}}$ $= 4x^{\frac{1}{3}}$ $= 4\sqrt[3]{x}$ **OR** By using indices $3x\sqrt[3]{x} = 3x \times x^{\frac{1}{3}}$ $= 3x^{\frac{4}{3}}$ Derivative $= 4\sqrt[3]{x}$	**2 marks – 1 for method** **1 for answer**
ii. $\frac{d}{dx}\left[\frac{\sin 2x}{e^{2x}}\right] = \frac{(e^{2x})(2\cos 2x) - (\sin 2x)(2e^{2x})}{(e^{2x})^2}$ $= \frac{2e^{2x}\left[\cos 2x - \sin 2x\right]}{(e^{2x})^2}$ $= \frac{2\left[\cos 2x - \sin 2x\right]}{e^{2x}}$	**2 marks – 1 for method** **1 for answer**
(b) i. $\int \frac{dx}{e^{3x}} = \int e^{-3x}dx = -\frac{1}{3}e^{-3x} + C$	**2 marks – 1 for method** **1 for answer**
ii. $\int_0^{\pi} \sec^2 \frac{x}{4}dx = 4\left[\tan\frac{x}{4}\right]_0^{\pi}$ $= 4\left[\tan\frac{\pi}{4} - \tan 0\right] = 4$	**2 marks – 1 for integral** **1 for answer**
(c) i. $\alpha + \beta = -\frac{b}{a} = -\frac{-4}{3} = \frac{4}{3}$ $2\alpha^2 + 2\beta^2 = 2\left(\alpha^2 + \beta^2\right)$ $= 2\left[(\alpha + \beta)^2 - 2\alpha\beta\right]$	**1 mark**
ii. $= 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]$ $= 2\left[\left(\frac{16}{9}\right) + \left(\frac{14}{3}\right)\right]$ $= \frac{116}{9}$	**1 mark**
iii. Equation with roots $2\alpha^2$ *and* $2\beta^2$ has equation $x^2 - (2\alpha^2 + 2\beta^2)x + (2\alpha^2 \times 2\beta^2) = 0$ i.e. $x^2 - 2[(\alpha + \beta)^2 - 2\alpha\beta]x + 4(\alpha\beta)^2 = 0$	**2 marks – 1 for method** **1 for answer**

$x^2 - 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]x + 4\left(\frac{-7}{3}\right)^2 = 0$ $x^2 - 2\left[\frac{16}{9} + \frac{14}{3}\right]x + \frac{196}{9} = 0$ $x^2 - 2\left[\frac{58}{9}\right]x + \frac{196}{9} = 0$ $x^2 - \frac{116}{9}x + \frac{196}{9} = 0$ $9x^2 - 116x + 196 = 0$	

Solutions	Marks/Comments
<u>Question 4</u>	
(a) i. $\angle FBD = \theta$ (Given) $\angle DBC = 90° - \theta$ $\angle BCD = 180° - 90° - (90 - \theta)$ (angle sum of ΔBCD) $= \theta$ $\therefore \angle FEA = \theta$ (Corresponding angles to $\angle BCD$, $FE \parallel BC$)	**2 marks – 1 for proof** **1 for reasons**
ii. $\angle AFE = 90°$ (Corresponding angles $FE \parallel BC$) $\tan\theta = \frac{AF}{y}$ $\therefore AF = y\tan\theta$	**1 mark**
iii. In ΔABD, $\cos\theta = \frac{z}{AF+FB}$ $z = (AF + FB)\cos\theta$ $= (x + y\tan\theta)\cos\theta$	**1 mark**
iv. $z = (x + y\tan\theta)\cos\theta$ $= x\cos\theta + y\tan\theta\cos\theta$ $= x\cos\theta + y\frac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}$ $= x\cos\theta + y\sin\theta$	**1 mark**
(b) $a = 500\ 000000 \times 0.8$ $r = \frac{4}{5}$ $S_\infty = \frac{a}{1-r}$ $= \frac{500\ 000\ 000 \times 0.8}{1-\frac{4}{5}}$ $= \frac{400\ 000\ 000}{\frac{1}{5}}$ \$2 000 000 000	**2 marks – 1 for substation into formula** **- 1 for answer**
(c) i. 	**1 for correct diagram**
ii. $AC^2 = 50^2 + 60^2 - 2 \times 50 \times 60\cos 120°$ AC = 95·3939.......... = 95 nm	**2 marks – 1 for substitution** **- 1 for answer**
$\frac{\sin\theta}{50} = \frac{\sin 120}{95.3939..}$ $\sin\theta = \frac{50\sin 120}{95.3939...}$ $\theta = 27°$ (nearest degree) Bearing $= 270 - 27$ $= 243°$	**2 marks – 1 for 27°** **- 1 for bearing**

Question 5

(a) i. α. P(Wins first Prize) = $\frac{1}{1000}$ — **1 mark**

β. P(At least \$500) = $\frac{2}{1000}$ $or\ 0\cdot002$ — **1 mark**

γ. P(no prizes) = $1-\frac{3}{1000}=\frac{997}{1000}$ $or\ 0\cdot997$ — **1 mark**

ii. P(At least \$500) = $1-(\frac{998}{1000}\times\frac{997}{999})$

$= 0\cdot003997997$ — **1 mark**

(b) ABCD is a parallelogram and BP = DQ

Then

AP = AB − PB

= DC − DQ

= QC

In Δ□s APD, BCQ

AP = QC (proven above)

AD = BC (opposite sides of parallelogram)

∠PAD = ∠QCB (opposites angles of a parallelogram)

$\therefore\ \Delta APD\ \equiv \Delta BCQ$ (SAS)

∴ DP = BQ (corresponding side in congruent triangles)

3 marks – 1 for showing AP = QC
1 for proving triangles congruent
1 for DP = BQ

(c) i. $\log 3+\log 9+\log 27+\ \dots\dots$

$\log 3+\log 3^2+\log 3^3+\ \dots\dots.$

log 3 + 2log 3 + 3log 3 + ...

Series is Arithmetic with a common difference of log 3

2 marks – 1 for type of series
1 for reason
i.e the value of d

ii. $S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{10}=\frac{10}{2}[2(\log_a 3)+(10-1)\log_a 3]$

$=5[2\log_a 3+9\log_a 3]$

$=5[11\log_a 3]$

$=55\log_a 3$

$=\log_a 3^{55}$ — **1 for S_{10} in either form**

(d) $4x^2-4x+4y^2+24y+21=0$

$x^2-x+y^2+6y=-\frac{21}{4}$

$\left(x^2-x+\frac{1}{4}\right)+(y^2+6y+9)=-\frac{21}{4}+\frac{1}{4}+9$

$\left(x-\frac{1}{2}\right)^2+(y+3)^2=4$ Centre $\left(\frac{1}{2},-3\right)$, Radius = 2

2 marks – 1 for centre
1 for radius

Question 6

(a) i. $\frac{dy}{dx} = 6(x-1)(x-2)$ $\qquad \frac{d^2y}{dx^2} = 12x - 18$

$= 6(x^2 - 3x + 2)$

$= 6x^2 - 18x + 12$

$y = \int(6x^2 - 18x + 12)\,dx$

$= 2x^3 - 9x^2 + 12x + C$

When $x = 1, y = 2$

$2 - 2(1)^3 - 9(1)^2 + 12(1) + C$

$2 = 2 - 9 + 12 + C$

$0 = 3 + C$

$C = -3$

Equation of curve is $y = 2x^3 - 9x^2 + 12x - 3$

2 marks – 1 for integration
1 for equation with correct value of "c"

ii. $\frac{dy}{dx} = 6(x-1)(x-2)$ but $\frac{dy}{dx} = 0$ for Stationary Points

i.e. $6(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

i.e. (1, 2), (2,1)

At (1, 2), $\frac{d^2y}{dx^2} < 0$ Maximum at (1, 2)

At (2, 1), $\frac{d^2y}{dx^2} > 0$ Mimimum at (2,1)

2 marks – 1 for points
1 for testing points

iii. $\frac{d^2y}{dx^2} = 12x - 18 = 0$ for inflexion point

$12x = 18$

$x = 1\frac{1}{2}$ i.e. $(1\frac{1}{2}, 1\frac{1}{2})$

At $(1\frac{1}{2}, 1\frac{1}{2})$

x	1	$1\frac{1}{2}$	2
$\frac{d^2y}{dx^2}$	$-$	0	$+$

Concavity changes at $x = 1\frac{1}{2}$

Point of inflexion at $(1\frac{1}{2}, 1\frac{1}{2})$

2 marks – 1 for inflexion point
1 for testing

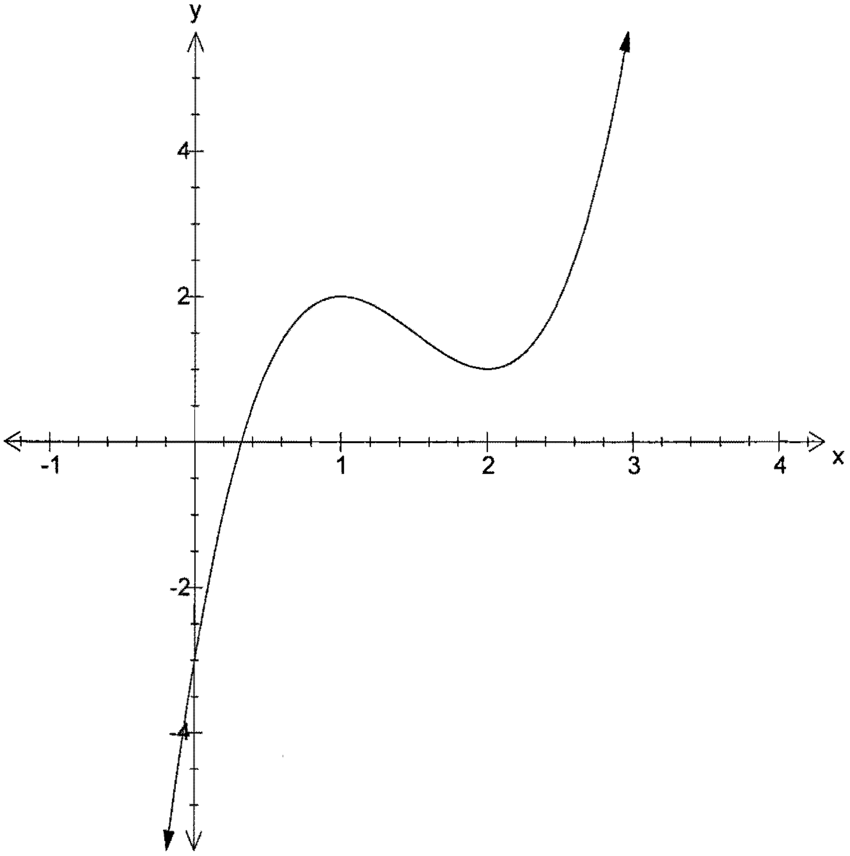
At $x = -1$, $y = 2(\ 1)^3 \quad 9(\ 1)^2 \ | \ 12(\ 1) \quad 3$

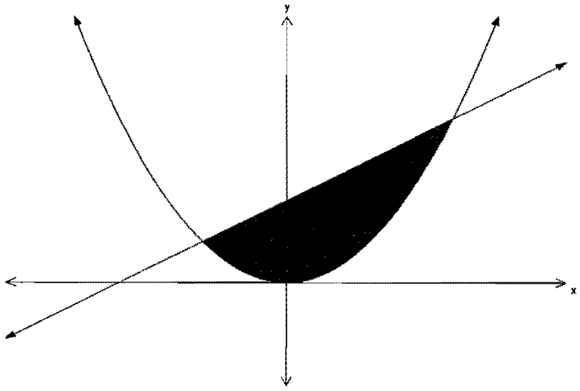
$= -26$

At $x = 3$, $y = 2(3)^3 - 9(3)^2 + 12(3) - 3$

$= 6$

At $x = 0$, $y = -3$

Solutions	Marks/Comments
	**2 marks – 1 for graph** **1 for labels**
(b) $\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}=\tan\theta$	**(3 marks)**
LHS $=\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}=\frac{\sec^2\theta\,.\cot\theta}{\text{cosec}^2\theta}$	1 mark
$=\frac{1}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}\div\frac{1}{\sin^2\theta}$	1 mark
$=\frac{1}{\cos\theta\sin\theta}\times\frac{\sin^2\theta}{1}$	
$=\frac{\sin\theta}{\cos\theta}$	1 mark
$=\tan\theta$ = RHS	
(c)	
$\lim_{\theta\to 0}\frac{\sin 2\theta}{3\theta}=\frac{2}{3}\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta}$	**1 mark**
$=\frac{2}{3}.1$	
$=\frac{2}{3}$	

Solutions	Marks/Comments
Question 7	
(a) $y = x^2 \quad (1)$ $y = x + 2 --- (2)$ *(1) In (2)* $x^2 = x + 2$ $x^2 - x - 2 = 0$ $(x + 1)(x - 2) = 0$ $x = -1$ or $x = 2$ *i.e.* $A(-1, 1)$ $B(2, 4)$  $A = \left\lvert\int_a^b (f(x) - g(x))\, dx\right\rvert$ $= \left\lvert\int_{-1}^{2} (x + 2 - x^2)\, dx\right\rvert$ $= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$ $= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$ $= 3\frac{1}{3} - -1\frac{1}{6}$ $= 4\frac{1}{2}$ *sq units*	**4 marks – 1 for each point (2)** **1 for integration** **1 for answer**
(b) i. Angle $= \frac{40}{60} \times 360$ $= 240 \times \frac{\pi}{180}$ $= \frac{4\pi}{3}$	**1 mark**
ii. $l = r\theta$ $= 12\left(\frac{4\pi}{3}\right)$ $= 16\pi$ cm.	**1 mark**
iii. $A = \frac{1}{2}r^2\theta$ $= \frac{1}{2}(12)^2\left(\frac{4\pi}{3}\right)$ $= 96\pi$ *cm.*	**1 mark**

(c) $A = \frac{h}{3}[ends + 2odds + 4\,evens]$

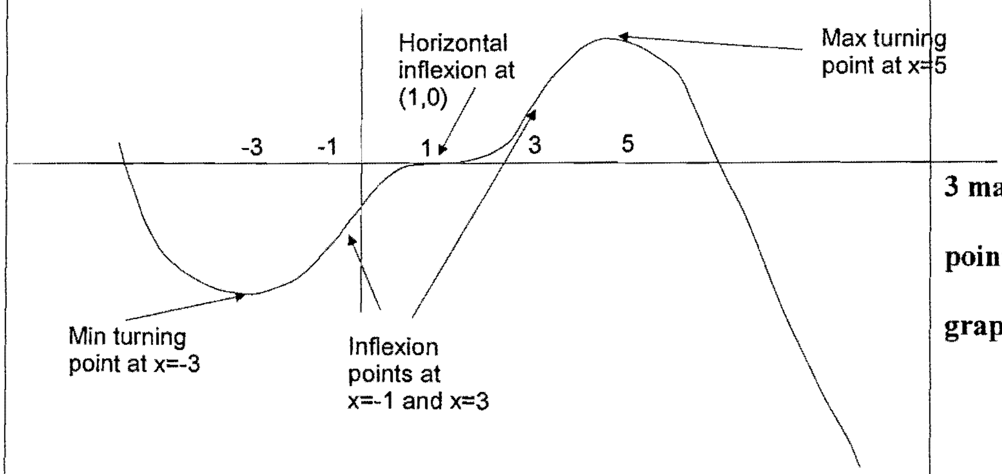
$= \frac{0.25}{3}\,[(3\cdot 43 + 1\cdot 97) + 2(0\cdot 38 + 2.65) + 4(2\cdot 17 + 1\cdot 87 + 2\cdot 31)]$

$= 3\cdot 071\dot{6}$

$= 3\cdot 1$ (1dp)

2 marks – 1 for use of formula
1 for answer

(d)



3 marks – 1 for inflexions
1 for stationary points
1 for point on graph (1, 0)

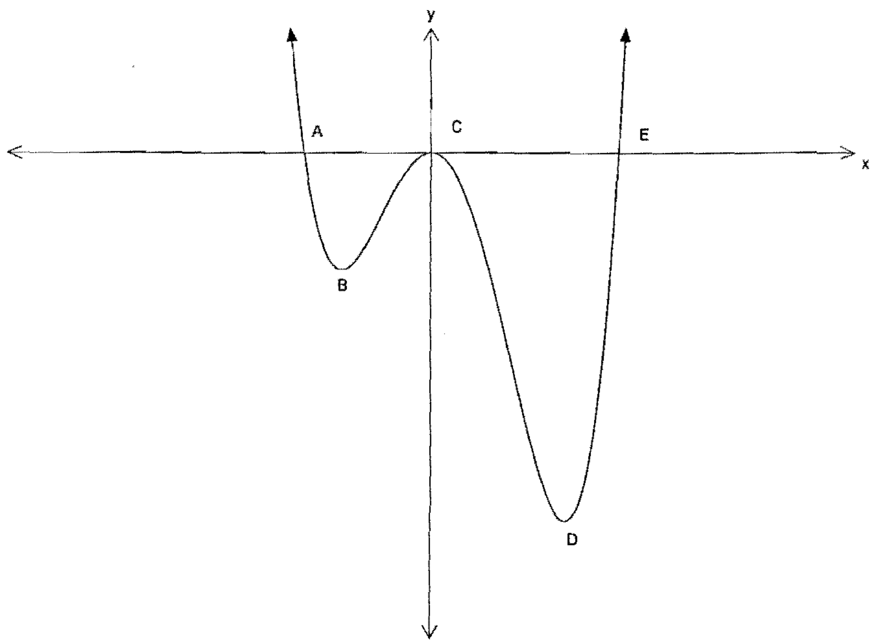
Solutions	Marks/Comments

Question 8

(a) i. B, C, D — **1 mark**

ii. From A to C and then from C to E — **1 mark**

iii.



1 mark

(b) i. Let \$P be the amount repaid each month

$\$A_n$ - Amount owing after n repayments

2 marks - 1 for working
1 for proof

$A_1 = 15\,000 \times 1{\cdot}005 - P$

$A_2 = A_1 \times 1\cdot005 - P$

$= (15\,000 \times 1{\cdot}005 - P) \times 1{\cdot}005 - P$

$= 15\,000 \times 1{\cdot}005^2 - P(1 + 1{\cdot}005)$

$A_3 = A_2 \times 1\cdot005 - P$

$= [15\,000 \times 1{\cdot}005^2 - P(1 + 1{\cdot}005)] \times 1.005 - P$

$= 15\,000 \times 1{\cdot}005^3 - P(1{\cdot}005 + 1{\cdot}005^2) - P$

$= 15\,000 \times 1{\cdot}005^3 - P(1 + 1{\cdot}005 + 1{\cdot}005^2)$

$= 15226{\cdot}13 - P(3{\cdot}015025)$

ii. $A_{24} = 15\,000 \times 1{\cdot}005^{24} - P(1 + 1{\cdot}005 + \ldots\ldots\ldots\ldots + 1{\cdot}005^{23})$

but $A_{24} = 10\,000$

3 marks - 1 for working
- 1 for sum of GS
- 1 for answer

$\therefore 10\,000 = 15\,000 \times 1\cdot005^{24} - P(1 + 1\cdot005 \ldots\ldots + 1\cdot005^{22})$

$P(1 + 1\cdot005 \ldots\ldots + 1\cdot005^{23}) = 15\,000 \times 1\cdot005^{24} - 10\,000$

$P = \dfrac{15\,000 \times 1{\cdot}005^{24} - 10\,000}{(1 + 1{\cdot}005 \ldots + 1{\cdot}005^{23})}$

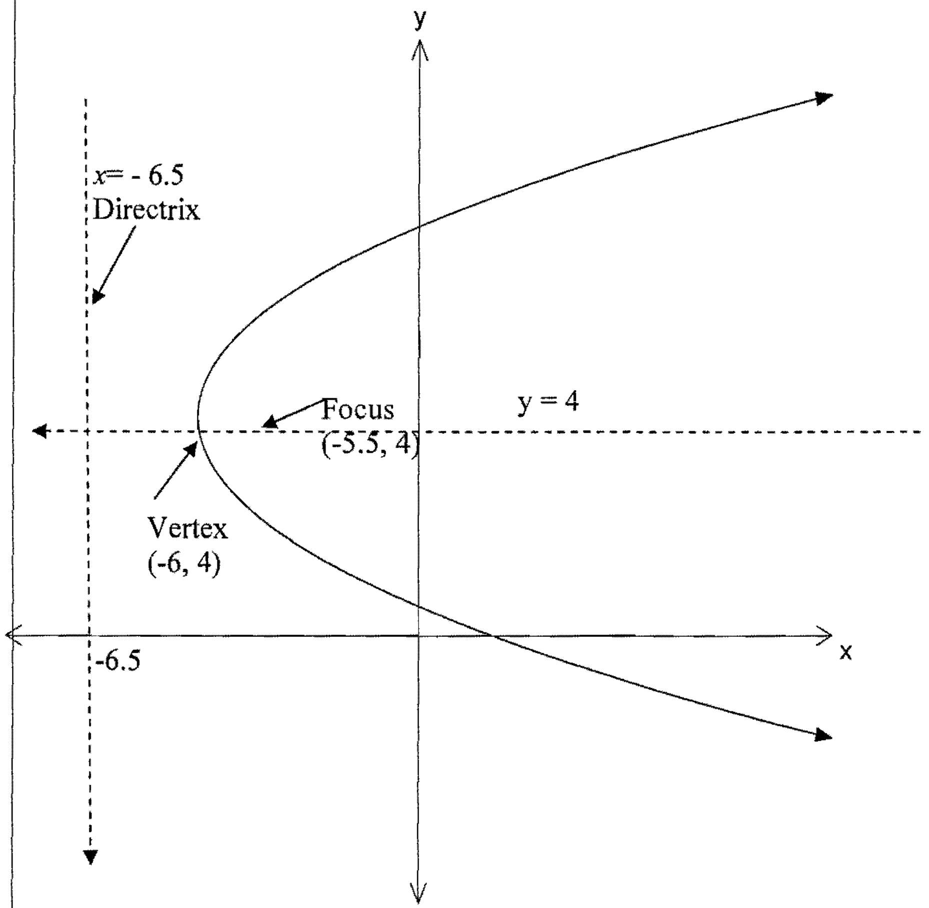
$= 15\,000 \times 1\cdot005^{24} - 10\,000 \div \dfrac{1{\cdot}005^{24} - 1}{0{\cdot}005}$

$= \dfrac{(15\,000 \times 1{\cdot}005^{24} - 10\,000) \times 0{\cdot}005}{1{\cdot}005^{24} - 1}$

GS with a = 1, r = 1·005, n = 24

$S = \dfrac{1(1{\cdot}005^{24} - 1)}{1{\cdot}005 - 1}$

$= \dfrac{1{\cdot}005^{24} - 1}{0{\cdot}005}$

$= \$271{\cdot}60$	
(c) $2x = y^2 - 8y + 4$ $y^2 - 8y = 2x - 4$ $y^2 - 8y + 16 = 2x - 4 + 16$ $(y-4)^2 = 2x + 12$ $(y-4)^2 = 2(x+6)$ Vertex $= (-6, 4)$ $4a = 2$, $a = \frac{1}{2}$ Focus $= \left(-5\frac{1}{2}, 4\right)$ Directrix : $x = -6\frac{1}{2}$ 	**4 marks – 1 for sketch** **1 for vertex** **1 for focus** **1 for directrix**

Solutions	Marks/Comments
Question 9	
(a) $x = 2\sin 2t$ $\dot{x} = 4\cos 2t$ $\ddot{x} = -8\sin 2t$	
i. $t = 0 \quad \dot{x} = 4\cos 2(0)$ $= 4 \times 1$ $= 4$ m / s	**1 mark**
ii. $t = \frac{\pi}{12} \quad \ddot{x} = 8\sin 2\left(\frac{\pi}{12}\right)$ $= -8\sin\left(\frac{\pi}{6}\right)$ $= -8 \times \frac{1}{2}$ $= -4$ m / s^2	**1 mark**
iii. $\dot{x} = 0$ *then* $4\cos 2t = 0$ i.e. $\cos 2t = 0$ $2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\ldots$ $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\ldots$	**2 marks – 1 for working 1 for answer**
iv. $x = 2\sin 2t$ $= 2\sin 2\left(\frac{\pi}{4}\right)$ $= 2$ $\therefore x = \pm 2$ m	**2 marks - 1 for working 1 for answer**
(b) $V = \int_a^b [f(x)]^2\, dx$ $= \int_1^3 \left(\sqrt{\frac{2x}{3x^2-1}}\right)^2 dx$ $= \int_1^3 \frac{2x\, dx}{3x^2-1}$ $= \frac{1}{3}[\ln(3x^2 - 1)]_1^3$ $= \frac{1}{3}[\ln(3 \times 3^2 - 1) - \ln(3 \times 1^2 - 1)]$ $= \frac{1}{3}(\ln 26 - \ln 2)$ $= \frac{1}{3}\left(\ln\frac{26}{2}\right)$ $= \frac{1}{3}(\ln 13)$	**3 marks – 1 use of formula 1 for Integral 1 for answer**

(c) i. $\frac{dV}{dt} = \frac{1}{100}(30t - t^2)$ when $t = 15$ $\frac{dV}{dt} = \frac{1}{100}[30(15) - (15)^2]$ $= \frac{225}{100}$ $= 2\frac{1}{4}$ cm^3 / min	**1 mark**
ii. $V = \int_0^{15} \frac{1}{100}(30t - t^2)$ $= \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right]_0^{15}$ $= \frac{1}{100}\left\{\left[15(15)^2 - \frac{15^3}{3}\right] - [0]\right\}$ $= \frac{1}{100}[3375 - 1125]$ $= \frac{1}{100}(2250)$ $= 22{\cdot}5$ cm^3	**2 marks – 1 for integral** **1 for answer**

Solutions	Marks/Comments
Question 10 (a) i. $SA = \pi r^2 + 2\pi r h = 300$ $2\pi r h = 300 - \pi r^2$ $h = \frac{300-\pi r^2}{2\pi r}$ $V = \pi r^2 h$ $= \pi r^2\left(\frac{300-\pi r^2}{2\pi r}\right)$ $= 150r - \frac{\pi r^3}{2}$	**2 marks – 1 for "h"** **1 for "V"**
ii. $V = 150r - \frac{1}{2}\pi r^3$ $\dot{V} = 150 - \frac{3}{2}\pi r^2$ $\ddot{V} = -3\pi r$ which is less than 0 for positive r Stat Pts when $\dot{V} = 0$ i.e. $150 - \frac{3}{2}\pi r^2 = 0$ $150 = \frac{3}{2}\pi r^2$ $100 = \pi r^2$ $r^2 = \frac{100}{\pi}$ $r^2 = \pm\sqrt{\frac{100}{\pi}}$ Now max Volume when $r > 0$ i.e. $r = \sqrt{\frac{100}{\pi}} = 5{\cdot}641895835$ $V = 150\sqrt{\frac{100}{\pi}} - \frac{\pi}{2}\left(\sqrt{\frac{100}{\pi}}\right)^3 = 564{\cdot}1895835$ $= 564\text{ m}^3$ (nearest m^3)	**4 marks – 1 for differentials** **1 for value of 'r'** **1 for test** **1 for max volume**
(b) i. $\frac{dP}{dt} = kP \quad \therefore P = P_0 e^{kt}$ When $t = 0$, $P = 20\,000$, $\therefore P_0 = 20\,000$ So $P = 20\,000e^{kt}$ When $t = 2$, $P = 25\,000$ $25\,000 = 20\,000e^{2k}$ $\frac{5}{4} = e^{2k}$ $\ln\left(\frac{5}{4}\right) = 2k$ $k = \ln\left(\frac{5}{4}\right) \div 2$ $k = 0{\cdot}111571775$ $\therefore P = 20\,000e^{0{\cdot}111571775\,t}$	**3 marks – 1 for value of 'k'** **1 for equation** **1 population**

When $t = 10$ $P = 20\,000e^{0\cdot111571775\,(10)}$ $= 61\,000$ people (nearest 100) ii. $\frac{dp}{dt} = 61\,000 \times 0\cdot111571775$. $= 6\,806$ people / year (c) $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$ $\log_a 2 + \log_a x^2 - \log_3 6 = \log_a 3$ $\text{Log}_a \frac{2x^2}{6} = \log_a 3$ $\therefore \frac{2x^2}{6} = 3$ $2x^2 = 18$ $x^2 = 9$ $x = \pm 3$ going back to original equation, cannot have log (-3) so $x = 3$	 **1 for rate of change** **2 marks – 1 manipulation of logs** **1 for answer**