HSC Seminars ~ Dubbo

Thursday 10th – Friday 11th June 2010

MATHS
Extension 1

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Preparing for the 2010 Mathematics Extension 1 HSC

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INTRODUCTION

EXAM STRATEGIES

USING THE STANDARDS PACKAGE— PRACTICE QUESTIONS

1. SYLLABUS OUTLINE

Mathematics 2/3 Unit Years 11-12 Syllabus. Board of Studies NSW (1982) p.6

Course Description (3 Unit)

The Mathematics 3 Unit Syllabus has been divided into a Preliminary course and an HSC course as follows:

Preliminary Course	HSC Course
Other inequalities (1.4 E)	Methods of integration (11.5)
Circle geometry (2.6–2.10)	Primitive of $\sin 2x$ and $\cos 2x$ (13.6 E)
Further trigonometry (sums and differences, t formulae, identities and equations) (5.6–5.9)	Equation $\frac{dN}{dt} = k(N - P)$ (14.2 E)
Angles between two lines (6.6)	Velocity and acceleration as a function of x (14.3 E)
Internal and external division of lines into given ratios (6.7 E)	Projectile motion (14.3 E)
Parametric representation (9.6)	Simple harmonic motion (14.4)
Permutations and combinations (18.1)	Inverse functions and inverse trigonometric functions (15.1–15.5)
Polynomials (16.1–16.3)	Induction (7.4)
Harder applications of the Preliminary 2 Unit course	Binomial theorem (17.1–17.3)
	Further probability (18.2)
	Iterative methods for numerical estimation of the roots of a polynomial equation (16.4)
	Harder applications of HSC 2 Unit topics (including 10.5 E, 13.4 E, 14.1 E)

2. DRAFT PERFORMANCE BANDS

MATHEMATICS EXTENSION 1

The typical performance in this band:

Band E4	• Exhibits extensive knowledge and skills appropriate to the Mathematics and Mathematics Extension 1 courses • Synthesises mathematical techniques, results and ideas creatively across the Mathematics and Mathematics Extension 1 courses to solve difficult problems • Uses sophisticated multi-step mathematical reasoning • Interprets, explains, justifies and evaluates solutions to problems • Translates efficiently between practical problems and their mathematical model • Communicates complex ideas and arguments effectively using appropriate mathematical language, notation, diagrams and graphs
Band E3	• Exhibits knowledge and skills appropriate to the Mathematics and Mathematics Extension 1 courses • Synthesises mathematical techniques, results and ideas from across the Mathematics and Mathematics Extension 1 courses to solve problems in areas such as geometry, calculus and probability • Uses multi-step mathematical reasoning such as mathematical induction • Translates between practical problems and their mathematical model in areas such as projectile motion • Communicates effectively using appropriate mathematical language, notation, diagrams and graphs
Band E2	• Exhibits knowledge of the techniques of the Mathematics and Mathematics Extension 1 courses • Uses techniques of integration from the Mathematics Extension 1 course such as integration by substitution • Uses logical reasoning in numerical contexts such as problems in algebra and geometry • Applies calculus to solve practical problems • Communicates using mathematical language, notation, diagrams and graphs
Band E1	

3. STANDARD INTEGRAL SHEET

Mathematics 2007 HSC Examination. Board of Studies NSW (2007)

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

4. USEFUL RESOURCES

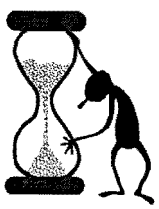
- Your mathematics teacher, any mathematics teacher, your friends, older brothers or sisters.
- School assessment task exams.
- Your own class notes and summaries.
- NSW Mathematics Extension 1 Stage 6 *Syllabus Outline* (see page 3 of this booklet).
- Board of Studies website www.boardofstudies.nsw.edu.au
  - Past HSC exam paper & Notes from the Marking Centre
  - Assessment Resource Centre
  - HSC examination timetable
- NSW HSC Standard Packages
  - 2002 Mathematics Extension 1
  - 2001 Mathematics Extension 1
- NSW HSC Online website www.hsc.csu.edu.au
  This website provides study strategies, tutorials in more than 40 HSC subjects, references, links to useful websites and other important materials for HSC students and teachers. It is hosted by Charles Sturt University and maintained by the NSW Department of Education and Training. The NSW Board of Studies and the Joint Council of NSW Professional Teachers' Associations are contributing partners.
- MANSW - Mathematical Association of NSW (Ask you teacher if they order copies through your school)
  - Green book of Past HSC 1999— 2000 (Mathematics 3 Unit Additional, 3/4 Unit Common), 2001— 2009 (Mathematics Extension 1) exam papers with worked solutions
  - "HSC Mathematics - Extension 1" *Practice Papers with Solutions* (M. Arnold)

  T (02) 9878 1487 **E** mansw@math.nsw.edu.au **W** www.hsc.csu.edu.au/pta/mansw/
- "Excel HSC Maths Extension 1" *Study Guide*
- "Cambridge - Mathematics Extension 1." *HSC Study Guide* (D. Arnold & G. Arnold)
- "Success One HSC Mathematics Extension 1" *HSC Study Guide*
- TEXTBOOK: "Maths in Focus 3 Unit " Books 1 and 2 (M. Grove)
- TEXTBOOK: "Cambridge 3 Unit Maths" Yr 11 (B. Pender)
- TEXTBOOK: "Cambridge 3 Unit Maths" Yr 12 (B. Pender)
- TEXTBOOK: "New Senior Maths 3 Unit" 11 & 12 (Fitzpatrick)
- TEXTBOOK: "3 Unit Maths Bk 1" (Jones & Couchman)
- TEXTBOOK: "3 Unit Maths Bk 2" (Jones & Couchman)

5. HSC EXAM INSTRUCTIONS

Mathematics Extension 1 2007 HSC Examination. Board of Studies NSW (2007)

- TOTAL MARKS = 84
- 7 Questions each worth 12 marks
- Reading Time - 5 minutes (i.e. work out your plan of attack!)
- Working time = 2 hours (I.e. 17 minutes per question!)
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of the paper (see page 3 of this booklet).



6. Examiners Comments from Past Extension 1 Mathematics HSCs

This is a summary of comments from the HSC markers of Extension 1 Mathematics. The comments are not in order of importance.

1. SHOW substitutions and working out clearly—marks are allocated for the PROCESS and the answer, not just the answer.
2. Watch the number of MARKS allocated to each part of the question— don't write 1/2 a page of explanation for 1 mark!
3. Take CARE—make sure you recall the formula CORRECTLY — marks lost from careless errors are as 'expensive' as marks lost by not doing the more difficult questions!
4. When PROVING a given result, check that your answer is the SAME!
5. CORRECT answers from INCORRECT working are not awarded marks— if you have made a mistake, fix the error in EVERY line of working.
6. LINK parts of a question—If (a) has parts (i) and (ii), often using the result in (i) makes answering (ii) easier.
7. Draw large, clear DIAGRAMS in pencil—about 1/3 a page is a good size.
8. Write working out CLEARLY—some students can't read their own writing, so markers have some difficulty awarding marks (i.e. 'r' & 'n' & 'x' often look similar, as do 5 & 8 and 4 & 9)
9. Read the question carefully — watch for the KEYWORDS e.g. 3 significant figures.
10. Use the table of STANDARD INTEGRALS.
11. Keep an eye on the TIME—don't spend so long on the early questions you don't get to Q7!
12. Remember Q2 is not as difficult as Q7—don't look for trouble!
13. If you make a mistake, just put a LINE through it—NEVER use liquid paper.
14. When using a CALCULATOR, don't round off too early and watch for radians/degrees.
15. Check that your answer is REASONABLE.

7. Tips for Success in the HSC Exam

1. Be prepared—do past HSC exams for practice so that you are familiar with the format of the exam and the types of questions asked.
2. Use the reading time and marking scheme to plan your approach to the exam.
3. Read the question carefully, noting any keywords. Choose a strategy to use and show all your working.
4. When you have completed a question, re-read it to make sure you have given the answer required.
5. Check that you have used the correct units and that your answer makes sense.
6. Attempt all questions—there are often 'easy' marks even in Q7!
7. At the end of the exam, check your work and go back and attempt harder or uncertain questions.

8. DEVELOPING AN EXAM TECHNIQUE

Adapted from: Robert Yen - Hurlstone Agricultural High School. 2001

WHY DEVELOP AN EXAM TECHNIQUE?

- Compared to other subjects, the questions and format of Maths exams are predictable.
- Getting used to doing exams and knowing what to expect saves time and worry.
- Getting into an exam "routine" keeps you focused on answering questions and managing your time wisely rather than panicking.

KNOW THE FORMAT OF YOUR HSC MATHS EXAM

- Time allowed .....
- Reading Time .....
- Number of sections .....
- Number of questions in each section .....
- Types of questions in each section .....
- Marks per section/question .....
- Total marks .....
- Recommended time per section .....
- Review time at end? .....

USE THE READING TIME TO PLAN YOUR EXAM

- How much can you do here without actually writing?
- Pre-read/skim the exam to see the work that is ahead of you. Familiarise yourself with the format of the exam and the answer booklets.
- Take note of the harder questions - these will require more time.
- Plan your time.

HOW EXAMINERS AWARD MARKS: THE IMPORTANCE OF SHOWING WORKING

- Principles of HSC marking: no half-marks, no marks for "bald" incorrect answers without working, sometimes no marks for "bald" correct answers if those answers could have been guessed. Marks can be awarded for correct working but wrong answer.
- Examiners award marks for successfully passing each stage of a solution. Once a mark has been awarded, it cannot be taken off.

9. HOW TO STUDY FOR MATHS

By: Robert Yen - Hurlstone Agricultural High School. 2001

Successful maths students take 4 steps, not just one...

1. PRACTISE YOUR MATHS

- Do your homework
- Master the basics
- Practise your maths skills thoroughly

2. REWRITE YOUR MATHS

- Rewrite the theory and examples in your own words
- Summarise each topic to gain a deeper understanding of the Maths course
- Your topic summaries may be written on sheets of paper or a separate book
- Mind maps may be drawn on large sheets
- Include important facts and formulas to be memorised, worked examples and personal reminders. Use diagrams, pictures, symbols and highlighting.
- Build up a collection of topic summaries to become your study notes.

3. ATTACK YOUR MATHS

- Identify the topics that require work
- Use your summaries for general revision, but use longer study periods for more in-depth work attacking your problem areas
- Can you work out your areas of weakness? Look at the course outline.
- Fill in any "gaps" in your Maths knowledge to get a bigger picture

4. TEST YOUR MATHS

- Test your understanding on past exams and mixed review exercises
- Learn from your mistakes - don't make them over and over again
- Use textbooks, revision books, study guides: revise
- Get used to doing HSC-type questions, work through past HSC papers
- Become familiar with the style, content and level of difficulty of HSC exams
- Practise exams under timed conditions
- Overcome exam anxiety and develop an exam technique
- Anticipate what will be asked in the exam: knowing what to expect minimises exam stress

In an exam, a *strong* student will read a question and know what to do straight away because they have practised that type of question in the past. A *weak* student, however, will be seeing that type of question for the first time here and waste much time thinking of and/or using strategies that may or may not work.

10. PRACTICE QUESTIONS— 2002 HSC QUESTION 1

2002 HSC Extension 1 Mathematics

(a) Evaluate $\lim_{x\to 0}\frac{\sin 3x}{x}$. 1

(b) Find $\frac{d}{dx}\left(3x^2\ln x\right)$ for $x>0$. 2

(c) Use the table of standard integrals to evaluate $\int_0^{\frac{\pi}{6}}\sec 2x\tan 2x\,dx$. 2

(d) State the domain and range of the function $f(x)=3\sin^{-1}\left(\frac{x}{2}\right)$. 2

(e) The variable point $\left(3t, 2t^2\right)$ lies on a parabola. Find the Cartesian equation for this parabola. 2

(f) Use the substitution $u=1-x^2$ to evaluate $\int_2^3\frac{2x}{\left(1-x^2\right)^2}dx$. 3

10. PRACTICE QUESTIONS— 2002 HSC QUESTION 3

2002 HSC Extension 1 Mathematics

(a) Seven people are to be seated at a round table.

(i) How many seating arrangements are possible? 1

(ii) Two people, Kevin and Jill, refuse to sit next to each other. How many seating arrangements are then possible? 2

(b) (i) Show that $f(x) = e^x - 3x^2$ has a root between $x = 3.7$ and $x = 3.8$. 1

(ii) Starting with $x = 3.8$, use one application of Newton's method to find a better approximation for this root. Write your answer correct to three significant figures. 3

(c) A household iron is cooling in a room of constant temperature 22°C. At time t minutes its temperature T decreases according to the equation

$$\frac{dT}{dt} = -k(T - 22) \text{ where } k \text{ is a positive constant.}$$

The initial temperature of the iron is 80°C and it cools to 60°C after 10 minutes.

(i) Verify that $T = 22 + Ae^{-kt}$ is a solution of this equation, where A is a constant. 1

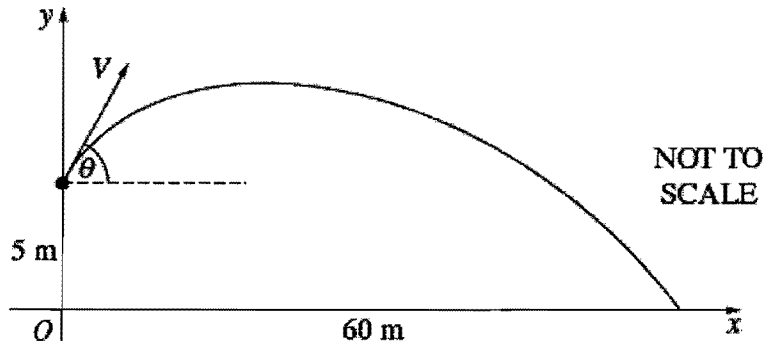
(ii) Find the values of A and k. 2

(iii) How long will it take for the temperature of the iron to cool to 30°C? Give your answer to the nearest minute. 2

10. PRACTICE QUESTIONS— 2002 HSC QUESTION 6(a)

2002 HSC Extension 1 Mathematics

(a)



An angler casts a fishing line so that the sinker is projected with a speed V m s^{-1} from a point 5 metres above a flat sea. The angle of projection to the horizontal is θ, as shown.

Assume that the equations of motion of the sinker are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10,$$

referred to the coordinate axes shown.

(i) Let (x, y) be the position of the sinker at time t seconds after the cast, and before the sinker hits the water. **2**

It is known that $x = Vt\cos\theta$.

Show that $y = Vt\sin\theta - 5t^2 + 5$.

(ii) Suppose the sinker hits the sea 60 metres away as shown in the diagram. **3**

Find the value of V if $\theta = \tan^{-1}\dfrac{3}{4}$.

(iii) For the cast described in part (ii), find the maximum height above sea level that the sinker achieved. **2**

10. PRACTICE QUESTIONS— 2002 HSC QUESTION 6(b)

2002 HSC Extension 1 Mathematics

(b) Let n be a positive integer.

(i) By considering the graph of $y=\frac{1}{x}$ show that 2

$$\frac{1}{n+1} < \int_n^{n+1} \frac{dx}{x} < \frac{1}{n}.$$

(ii) Hence deduce that 3

$$\left(1+\frac{1}{n}\right)^n < e < \left(1+\frac{1}{n}\right)^{n+1}.$$

End of Question 6